MTH 317/617
Homework #8

Due Date: November 20, 2023

1 Problems for Everyone

1. For the following functions find the first four terms of the Taylor series about z_0 and determine the radius of convergence of the series

   (a) $\dfrac{1}{1+z}$, $z_0 = 0$.

   (b) e^{-z^2}, $z_0 = 0$.

   (c) $z^3 \sin(3z)$, $z_0 = 0$.

   (d) $z^3 \sin(3z)$, $z_0 = 0$.

   (e) $\dfrac{1+z}{1-z}$, $z_0 = i$.

   (f) $\dfrac{e^z}{3-2z}$, $z_0 = 0$.

   (g) $\dfrac{z}{(1-z)^2}$, $z_0 = 0$.

2. pg. 212, #3

3. pg. 212, #4

4. pg. 212, #6, follow the problem's hint and use Taylor's theorem to expand the numerator for each integral.

5. Find the first four terms of the Laurent series for the function $f(z) = \dfrac{1}{z+z^2}$ in each of the following domains

   (a) $0 < |z| < 1$

   (b) $|z| > 1$

   (c) $0 < |z+1| < 1$

   (d) $1 < |z+1|$

6. Find the first four terms of the Laurent series for the following functions about the indicated point

   (a) $\dfrac{e^z - 1}{z^2}$; $z_0 = 0$

   (b) $\dfrac{z^2}{z^2 - 1}$; $z_0 = 1$

   (c) $\dfrac{\sin(z)}{(z - \pi)^2}$; $z_0 = \pi$

   (d) $\dfrac{z}{(\sin(z))^2}$; $z_0 = 0$

   (e) $\dfrac{1}{e^z - 1}$; $z_0 = 0$

7. Evaluate the following contour integrals:

   (a) $\displaystyle\int_{|z|=1} \frac{z^2 + 3z - 1}{z(z^2 - 3)} dz$

   (b) $\displaystyle\int_{|z|=1} \frac{\sin(z)}{z^6} dz$

   (c) $\displaystyle\int_{|z|=4} z \tan(z) dz$

   (d) $\displaystyle\int_{|z|=1} \frac{e^{z^2}}{z^6} dz$

   (e) $\displaystyle\int_{|z|=1} z^4 (e^{z^{-1}} + z^2) dz$

   (f) $\displaystyle\int_{|z|=1} \cos\left(\frac{1}{z^2}\right) e^{z^{-1}} dz$

   (g) $\displaystyle\int_{|z|=1} \frac{1}{z^2}(e^z - 1) dz$