

MTH 317/617
Quiz #4

1. Let $f : \mathbb{C} \mapsto \mathbb{C}$ be defined by

$$e^{z^3} = e^{x^3 - 3xy^2} (\cos(3x^2y - y^3) + i \sin(3x^2y - y^3))$$

and define the real valued function $u : \mathbb{R}^2 \mapsto \mathbb{R}$ by

$$u(x, y) = \text{Im}(f(z)).$$

(a) **Short Answer:** Explain why u is a harmonic function.

Since f is analytic, $\text{Im}(f(z))$ is harmonic.

(b) Find a function $v : \mathbb{R}^2 \mapsto \mathbb{R}$ such that $u(x, y) + i(v(x, y))$ is analytic.

Since $\text{Im}(f(z)) = \text{Re}(-if(z))$ it follows that

$$\begin{aligned} v &= \text{Im}(-if(z)) \\ &= -e^{x^3 - 3xy^2} \cos(3x^2y - y^3) \end{aligned}$$