Homework 1

Analysis

Due: January 22, 2018

1. Give an $\varepsilon - \delta$ proof that for all $x \in [-1, 1]$:
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}.$$
2. Prove that for all $x, y, z \in \mathbb{R}$:
$$|x - y| \geq \left||x - z| - |z - y|\right|.$$
3. Let a_n be a sequence in $\mathbb{R}$ satisfying for all $n \in \mathbb{N}$
$$|a_{n+1} - a_n| < \frac{1}{n}.$$
Is a_n a Cauchy sequence? Prove or give a counterexample.
4. Let a_n be a sequence in $\mathbb{R}$ satisfying for all $n \in \mathbb{N}$
$$|a_{n+1} - a_n| < \frac{1}{n^2}.$$
Is a_n a Cauchy sequence? Prove or give a counterexample.
5. Let a_n be a sequence in $\mathbb{R}$. Suppose there are numbers k, $0 < k < 1$ and $C > 0$ such that
$$|x_{n+1} - x_n| \leq Ck^n$$
for all $n \geq 1$. Is a_n a Cauchy sequence? Prove or give a counterexample.
6. Define a sequence of real numbers by $x_1 = 2$ and
$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}.$$
Prove that x_n is Cauchy. **Hint:** The result of problem number 5 could be useful here.