Quiz 4

Analysis

February 23, 2018

1. Let (X, d) be a metric space, and let $f : X \mapsto (\mathbb{R}^2, \|\cdot\|_2)$ and $g : X \mapsto (\mathbb{R}^2, \|\cdot\|_2)$ be continuous mappings with respect to this metric. Let $h : X \mapsto \mathbb{R}^4$ be a mapping defined by

$$h(x) = (f(x), g(x)).$$

Prove that h is a continuous mapping from (X, d) to $(\mathbb{R}^4, \|\cdot\|_2)$.

Suppose $x_n \to x$ in X. Therefore,

$$\lim_{n\to\infty} \|h(x_n) - h(x)\|_2 = \lim_{n\to\infty} \left(\|f(x_n) - f(x)\|_2^2 + \|g(x_n) - g(x)\|_2^2\right)^{1/2}$$

$$= \left(\lim_{n\to\infty} \|f(x_n) - f(x)\|_2^2 + \lim_{n\to\infty} \|g(x_n) - g(x)\|_2^2\right)^{1/2}$$

$$= 0.$$