Quiz 6

Analysis

March 14, 2018

1. Suppose that $f(t, u)$ is a Lipschitz continuous function of u, uniformly in t. Use Gronwall's inequality to prove that solutions to the initial value problem

$$\frac{du}{dt} = f(t, u) \qquad \text{✳}$$
$$u(0) = u_0$$

are unique.

Let U, V solve (✳). Then,

$$\frac{du}{dt} - \frac{dv}{dt} = f(t, u) - f(t, v)$$

$$\Rightarrow U(t) - V(t) \leq \int_0^t \big(f(s, u) - f(s, v)\big) ds$$

$$\Rightarrow |U(t) - V(t)| \leq \int_0^t |f(s, u) - f(s, v)| ds$$

$$\leq C \int_0^t |U(s) - V(s)| ds$$

By Gronwall's inequality:

$$|U(t) - V(t)| \leq 0 \, e^{Ct} = 0.$$