Homework #3.

pg. 442, #2

$$y' - y = 2e^t, \quad y_0 = 3$$

$$\Rightarrow \mathcal{L}[y'] - \mathcal{L}[y] = 2\mathcal{L}[e^t]$$

$$\Rightarrow p\mathcal{L}[y] - 3 - \mathcal{L}[y] = 2\int_0^\infty e^t e^{-pt}\,dt$$

$$\Rightarrow (p-1)\mathcal{L}[y] = \frac{2}{p-1} + 3$$

$$\Rightarrow \mathcal{L}[y] = \frac{2}{(p-1)^2} + \frac{3}{(p-1)}$$

$$\Rightarrow y = 2te^t + 3e^t$$

pg. 443, #16

$$y'' + 9y = \cos(3t), \quad y_0 = 2, \ y_0' = 0$$

$$\Rightarrow (p^2+9)\mathcal{L}[y] - 2p - 18 = \frac{p}{9+p^2}$$

$$\Rightarrow \mathcal{L}[y] = \frac{p}{(9+p^2)^2} + \frac{2p}{p^2+9} + \frac{18}{p^2+9}$$

$$\Rightarrow \mathcal{L}[y] = \frac{1}{6}\,\frac{2\cdot 3\cdot p}{(3^2+p^2)^2} + 2\,\frac{p}{p^2+3^2} + 18\,\frac{?}{p^2+3^2}$$

$$\Rightarrow y = \frac{1}{6}\,t\sin(3t) + 2\cos(3t)$$

pg. 448, #1

$$g * h = \int_{-\infty}^{\infty} g(t-\tau)h(\tau)\,d\tau = \int_{-\infty}^{\infty} g(u)h(t-u)\,du = h * g.$$

pg. 448, #2

$$\mathcal{L}^{-1}\left[\frac{1}{(p+a)(p+b)}\right] = e^{-at} * e^{-bt}$$

$$= \int_0^t e^{-a(t-\tau)} e^{-b\tau} d\tau$$

$$= e^{-at} \int_0^t e^{(a-b)\tau} d\tau$$

$$= \frac{e^{-at}}{a-b} e^{(a-b)\tau}\Big|_0^t$$

$$= \frac{1}{a-b} e^{-at}\left(e^{(a-b)t} - 1\right)$$

$$= \frac{1}{a-b} e^{-bt} - e^{-at}$$

$$= \frac{e^{-at} - e^{-bt}}{b-a}.$$

pg. 448, #8

$$\mathcal{L}^{-1}\left[\frac{1}{(p+a)(p+b)(p+c)}\right] = e^{-at} * \left(\frac{e^{-bt} - e^{-ct}}{c-b}\right)$$

$$= \int_0^t e^{-a(t-\tau)} \frac{e^{-b\tau} - e^{-c\tau}}{c-b} d\tau$$

$$= \frac{e^{-at}}{(a-b)(a-c)} + \frac{e^{-bt}}{(b-c)(b-a)} + \frac{e^{-ct}}{(c-a)(c-b)}$$