

# Quiz 1

MST 306

January 31, 2019

1. Find the Fourier transform of the triangular pulse:

$$f(x) = \begin{cases} h(1 - a|x|), & |x| < \frac{1}{a} \\ 0, & |x| > \frac{1}{a} \end{cases}$$

where  $h, a > 0$  are constants.

$$\begin{aligned} \mathcal{F}[f](\alpha) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx \\ &= \frac{1}{\pi} \int_0^{1/a} h(1 - ax) \cos(\alpha x) dx \\ &= \frac{1}{\pi} \int_0^{1/a} h \cos(\alpha x) - \frac{ha}{\pi} \int_0^{1/a} x \cos(\alpha x) dx \\ &= \frac{h}{\alpha\pi} \sin\left(\frac{\alpha}{a}\right) - \frac{ha}{\pi} \left[ \frac{x \sin(\alpha x)}{\alpha} \Big|_0^{1/a} - \int_0^{1/a} \frac{\sin(\alpha x)}{\alpha} dx \right] \\ &= \frac{h}{\alpha\pi} \sin\left(\frac{\alpha}{a}\right) - \frac{h}{\pi\alpha} \sin\left(\frac{\alpha}{a}\right) + \frac{ha}{\pi} \frac{\cos(\alpha x)}{\alpha^2} \Big|_0^{1/a} \\ &= \frac{ha}{\pi\alpha^2} \left( 1 - \cos\left(\frac{\alpha}{a}\right) \right) \end{aligned}$$