MST 205 Homework 10 Solutions

page 267 problem 2:

(2) $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5x_1 + 3x_2 \\ -6x_1 - 4x_2 \end{bmatrix}$, $\beta = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$, $v = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

1pt (a) $[T]_\alpha^\alpha = \left[[T(e_1)]_\alpha \mid [T(e_2)]_\alpha \right] = \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}$

1pt (b) $[I]_\beta^\alpha = \left[\begin{bmatrix} 2 \\ 1 \end{bmatrix}_\alpha \mid \begin{bmatrix} 1 \\ 1 \end{bmatrix}_\alpha \right] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

1pt (c) $[I]_\alpha^\beta = ([I]_\beta^\alpha)^{-1}$

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right] \Rightarrow [I]_\alpha^\beta = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

1pt (d) $[T]_\beta^\beta = [I]_\alpha^\beta [T]_\alpha^\alpha [I]_\beta^\alpha = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 29 & 18 \\ -45 & -28 \end{bmatrix}$

1pt (e) $\left[[I]_\alpha^\beta \mid v \right] = \left[\begin{array}{cc|c} 2 & 1 & 5 \\ 1 & 1 & -4 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & -13 \end{array}\right] \Rightarrow [v]_\beta = \begin{bmatrix} 9 \\ -13 \end{bmatrix}$

1pt (f) $[T(v)]_\beta = [T]_\beta^\beta [v]_\beta = \begin{bmatrix} 29 & 18 \\ -45 & -28 \end{bmatrix}\begin{bmatrix} 9 \\ -13 \end{bmatrix} = \begin{bmatrix} 27 \\ -41 \end{bmatrix}$

page 277 problem 2:

2pt (2) $\begin{bmatrix} 4 & -4 \\ 1 & 0 \end{bmatrix}$ $\xrightarrow{\det(A - \lambda I)}$

$$\det \begin{bmatrix} 4-\lambda & -4 \\ 1 & -\lambda \end{bmatrix} = (4-\lambda)(-\lambda) - (-4)(1) = \lambda^2 - 4\lambda + 4 = (\lambda - 2)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 2$$

$$A - \lambda I = \begin{bmatrix} 4-2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \Rightarrow \begin{array}{l} 2x_1 - 4x_2 = 0 \\ x_1 - 2x_2 = 0 \end{array} \Rightarrow \begin{array}{l} x_1 = 2 \\ x_2 = 1 \end{array} \Rightarrow E(\lambda) = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$$

page 377 problem 6:

Sol (6) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\det(A-\lambda I) = \det\begin{bmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix} = (1-\lambda)(4-\lambda)-(2)(3) = \lambda^2-5\lambda-2 = 0$$

$$\Rightarrow \lambda_{1,2} = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$$

$$A-\lambda_1 I = \begin{bmatrix} 1-\left(\frac{5}{2}+\frac{\sqrt{33}}{2}\right) & 2 \\ 3 & 4-\left(\frac{5}{2}+\frac{\sqrt{33}}{2}\right) \end{bmatrix} \Rightarrow E(\lambda_1) = \operatorname{span}\left\{\begin{bmatrix} -3+\sqrt{33} \\ 6 \end{bmatrix}\right\}$$

$$A-\lambda_2 I = \begin{bmatrix} 1-\left(\frac{5}{2}-\frac{\sqrt{33}}{2}\right) & 2 \\ 3 & 4-\left(\frac{5}{2}-\frac{\sqrt{33}}{2}\right) \end{bmatrix} \Rightarrow E(\lambda_2) = \operatorname{span}\left\{\begin{bmatrix} -3-\sqrt{33} \\ 6 \end{bmatrix}\right\}$$