pg. 277, #14

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

$\lambda_1 = 1$

$$\det\left(\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} - \lambda I\right) = \det\begin{pmatrix} -\lambda & 2 \\ 1 & -\lambda \end{pmatrix} = \lambda^2 - 2 = 0$$

$$\Rightarrow \lambda = \pm\sqrt{2}$$

$\lambda_2 = \sqrt{2}$

$$\begin{bmatrix} -\sqrt{2} & 2 \\ 1 & -\sqrt{2} \end{bmatrix} \Rightarrow x = \tfrac{2}{\sqrt{2}}y = \sqrt{2}y.$$

$\lambda_3 = -\sqrt{2}$

$$\begin{bmatrix} \sqrt{2} & 2 \\ 1 & \sqrt{2} \end{bmatrix} \Rightarrow x = -\sqrt{2}y$$

Eigenvectors: [illegible]

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \cancel{0}\ \sqrt{2}+3 \\ \sqrt{2} \\ 1 \end{bmatrix}, \begin{bmatrix} \cancel{0} \\ -\sqrt{2} \\ 1 \end{bmatrix} -\sqrt{2}+3$$

pg. 286, #8

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

→ eigenvalues = 1, 1, 2

→ find eigenvectors...

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} [illegible] \\ -1 \\ 1 \end{bmatrix} -1$$

GM ≠ AM ∴ not diagonalizable

~~pg.~~ 286, #22

$$p(\lambda) = \lambda^5 - 2\lambda^4 + \lambda^3$$
$$= \lambda^3(\lambda^2 - 2\lambda + 1)$$
$$= \lambda^3(\lambda - 1)^2$$

$$\begin{bmatrix} 0 & 0 & & & \\ & 0 & 0 & & \\ & & 0 & & \\ & & & 1 & 0 \\ & & & & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & & & \\ & 0 & 0 & & \\ & & 0 & & \\ & & & 1 & 0 \\ & & & & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & & \\ & & & 1 & 0 \\ & & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & & & \\ & 0 & 0 & & \\ & & 0 & & \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & & \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}$$

~~pg.~~ 286, #33

$$A = PBP^{-1}$$
$$A^k = PB\cancel{P^{-1}}\,\cancel{P}B\cancel{P^{-1}}\,\cancel{P}BP^{-1}\cdots\cancel{P}BP^{-1}$$
$$= PB^kP^{-1}$$

~~pg.~~ 286, #34

$$A = PBP^{-1}$$
$$A^{-1} = (PBP^{-1})^{-1}$$
$$= P^{-1}B^{-1}P$$
$$\Rightarrow B^{-1} = PA^{-1}P^{-1}$$

This works since

$$B^{-1}B = PA^{-1}\cancel{P^{-1}}\cancel{P}AP^{-1}$$
$$= PA^{-1}AP^{-1}$$
$$= PP^{-1}$$
$$= I$$