Homework #4

pg. 73, #2.

P.6.

$$(c+d)\odot x = x^{c+d}$$
$$= x^c x^d$$
$$= (c\odot x)(d\odot x)$$
$$= (c\odot x)\oplus(d\odot x).$$

P.7.

$$c\odot(d\odot x) = c\odot x^d$$
$$= (x^d)^c$$
$$= x^{dc}$$
$$= (x^c)^d$$
$$= d\odot x^c$$
$$= d\odot(c\odot x).$$

P.8

$$1\odot x = x^1$$
$$= x.$$

pg. 81, #2.

a) yes
b) no
c) yes
d) no.

pg. 82, #12.

$$\begin{bmatrix} 3 & 5 \\ 3 & 4 \end{bmatrix} = c_1\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + c_2\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + c_3\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{array}{l} c_1+c_2+c_3=3 \\ c_1+c_2=5 \\ c_2=3 \\ c_1-c_3=4 \end{array} \Rightarrow \begin{array}{l} c_1+c_3=0 \\ c_1=2 \\ c_1-c_3=4 \end{array} \Rightarrow \begin{array}{l} c_3=-2 \\ c_3=-2 \end{array}$$

Therefore, $\begin{bmatrix} 3 & 5 \\ 3 & 4 \end{bmatrix}$ is in the span of these matrices.

pg. 82, #16.

We just need to check linear dependence.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 0 & -1 & 1 \end{bmatrix} \begin{matrix} \\ -R1 \\ \\ \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & -1 & 1 \end{bmatrix} \begin{matrix} \\ \\ +R2 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, these vectors are linearly ind. and cannot span $\mathbb{R}^3$.

pg. 82, #20.

They cannot span $P_3$ since $\dim(P_3)=4$.