Homework #6

*pg. 104, #6

$$\left[\begin{array}{cc|c|c} -2 & 4 & 0 & a \\ 1 & -2 & 0 & b \end{array}\right] +2R2 \rightarrow \left[\begin{array}{cc|c|c} 0 & 0 & 0 & b+2a \\ 1 & -2 & 0 & b \end{array}\right]$$

Nullspace:

$x_1 = 2x_2$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow NS(A) = span\left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right\}.$$

Columnspace

$b = -2a$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ -2a \end{bmatrix} = a \begin{bmatrix} 1 \\ -2 \end{bmatrix} \Rightarrow CS(A) = span\left\{\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}.$$

Rowspace

$$RS(A) = span\left\{\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}.$$

*pg 104, #10

$$\left[\begin{array}{cccc|c|c} 2 & -1 & 3 & 4 & 0 & a \\ 1 & 0 & -1 & 3 & 0 & b \end{array}\right] -2R2 \Rightarrow \left[\begin{array}{cccc|c|c} 0 & -1 & 5 & -2 & 0 & a-2b \\ 1 & 0 & -1 & 3 & 0 & b \end{array}\right]$$

Nullspace:

$x_2 = 5x_3 - 2x_4$

$x_1 = x_3 - 3x_4$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_3 - 3x_4 \\ 5x_3 - 2x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

$$NS(A) = span\left\{\begin{bmatrix} 1 \\ 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}\right\}$$

Columnspace:

$$CS(A) = span\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}.$$

Rowspace:

$$RS(A) = span\left\{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 5 \\ -2 \end{bmatrix}\right\} \Rightarrow Rank(A) = 2.$$

* pg. 105, #18

If $m > n$, then when row reducing you get a row of zeros which implies the rows are linearly dependent.

pg. 110, #2

$f(x) = \cos(5x)$, $g(x) = \sin(5x)$

$f'(x) = -5\sin(5x)$, $g'(x) = 5\cos(5x)$

$$\text{Det}\begin{bmatrix} f & g \\ f' & g' \end{bmatrix} = 5\cos^2(5x) + 5\sin^2(5x) = 5 \neq 0.$$

Therefore, f, g are linearly ind.

pg. 110, #12.

$f(x) = \sin^2(x)$, $g(x) = \cos^2(x)$, $h(x) = \cos(2x) = \cos^2(x) - \sin^2(x)$

Therefore,

$h(x) = g(x) - f(x)$

and thus f, g, h are linearly dependent.