

MST 205

Homework #6

Due Date: March 25, 2022

1. In the following problems sketch the solution curves as functions of time t for the following differential equations. Be sure to calculate any inflection points and make sure your solution curves change concavity at the correct points.

(a) $\frac{dx}{dt} = 4x^2 - 16$

(b) $\frac{dx}{dt} = x - x^3$

(c) $\frac{dx}{dt} = 1 + \frac{1}{2} \cos(x)$

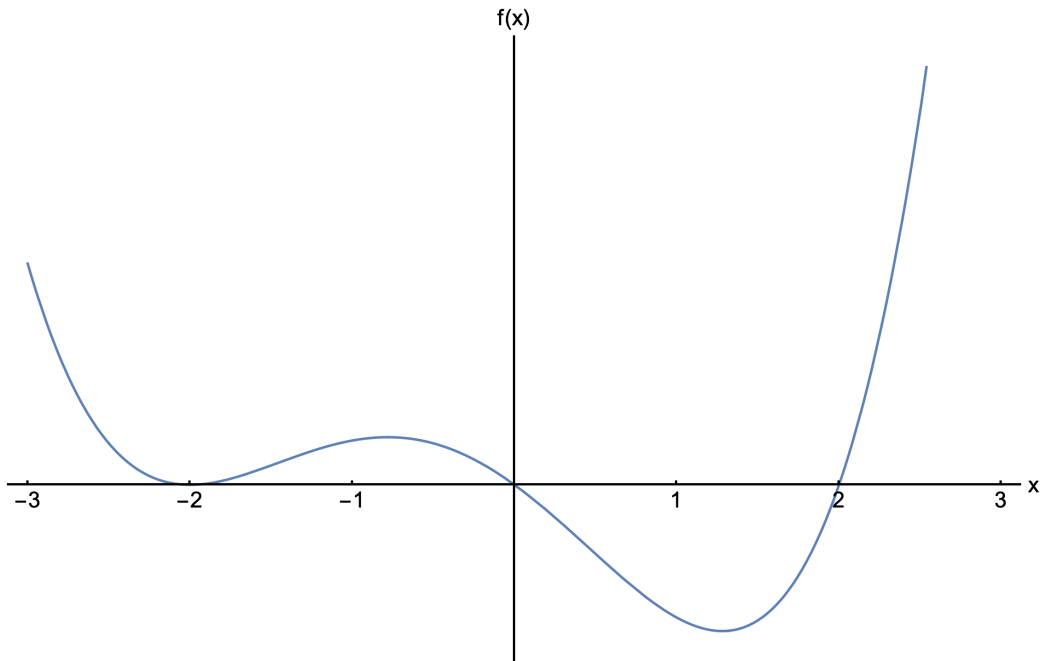
(d) $\frac{dx}{dt} = 1 - 2 \cos(x)$

(e) $\frac{dx}{dt} = e^{-x} \sin(x)$

2. Consider the differential equation

$$\frac{dx}{dt} = f(x),$$

where $f(x)$ is plotted below.



- (a) On the figure indicate any fixed points, i.e. equilibrium points, for this differential equation.
- (b) On one axis, sketch the corresponding solutions curves $x(t)$ for this problem. Your solution curves should contain all possible qualitatively different types of solution curves.

3. The curves $x(t)$ illustrated below correspond to solution curves for the differential equation $\frac{dx}{dt} = f(x)$.

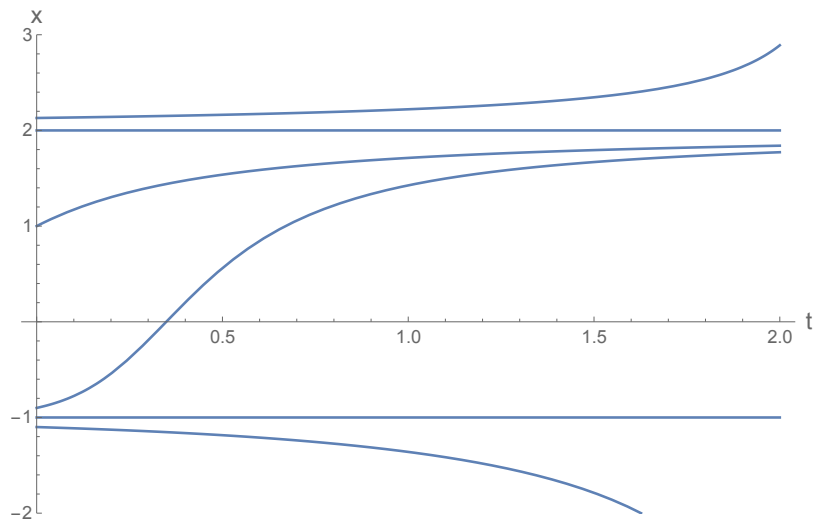


Figure 1:

- (a) Sketch a graph of $f(x)$ that is consistent with the above figure.
- (b) Give a formula for $f(x)$ that is consistent with the above figure.
4. For each of (a)-(d) find an equation $\frac{dx}{dt} = f(x)$ with the stated properties, or if there are no examples, explain why not. In each problem, assume that f is a smooth function, i.e. infinitely differentiable.
- (a) Every real number is a fixed point.
- (b) Every integer is a fixed point, and there are no others.
- (c) There are no fixed points.
- (d) There are precisely 100 fixed points.