MST 205
Homework #6

Due Date: March 25, 2022

1. In the following problems sketch the solution curves as functions of time t for the following differential equations. Be sure to calculate any inflection points and make sure your solution curves change concavity at the correct points.

   (a) $\dfrac{dx}{dt} = 4x^2 - 16$

   (b) $\dfrac{dx}{dt} = x - x^3$

   (c) $\dfrac{dx}{dt} = 1 + \dfrac{1}{2}\cos(x)$

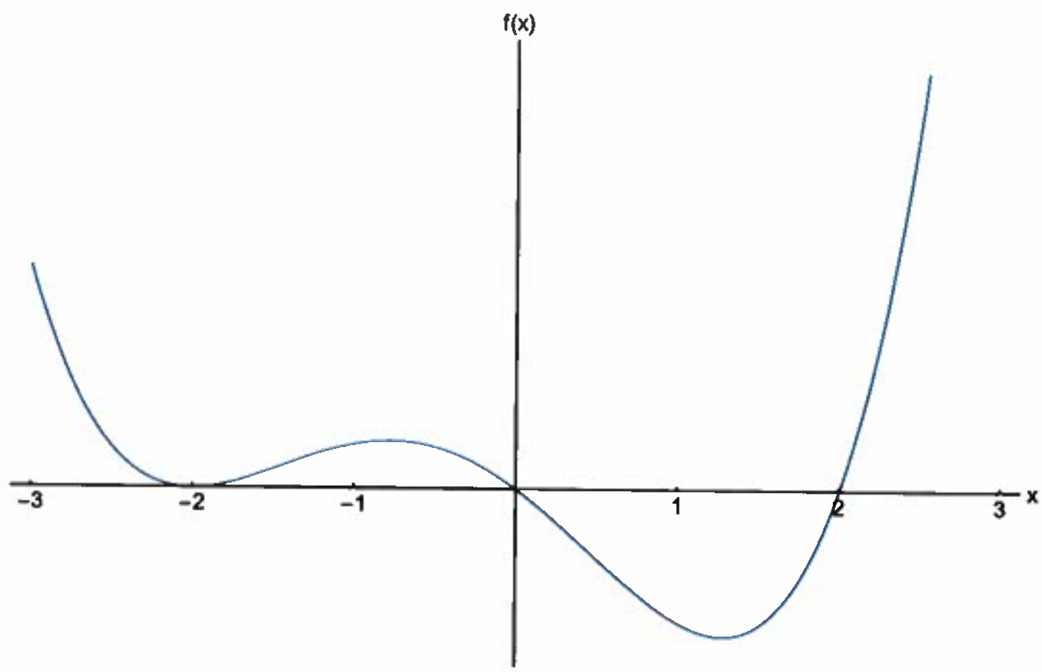
   (d) $\dfrac{dx}{dt} = 1 - 2\cos(x)$

   (e) $\dfrac{dx}{dt} = e^{-x}\sin(x)$

2. Consider the differential equation

$$\frac{dx}{dt} = f(x),$$

where $f(x)$ is plotted below.



   (a) On the figure indicate any fixed points, i.e. equilibrium points, for this differential equation.

   (b) On one axis, sketch the corresponding solutions curves $x(t)$ for this problem. Your solution curves should contain all possible qualitatively different types of solution curves.

3. The curves $x(t)$ illustrated below correspond to solution curves for the differential equation $\frac{dx}{dt} = f(x)$.

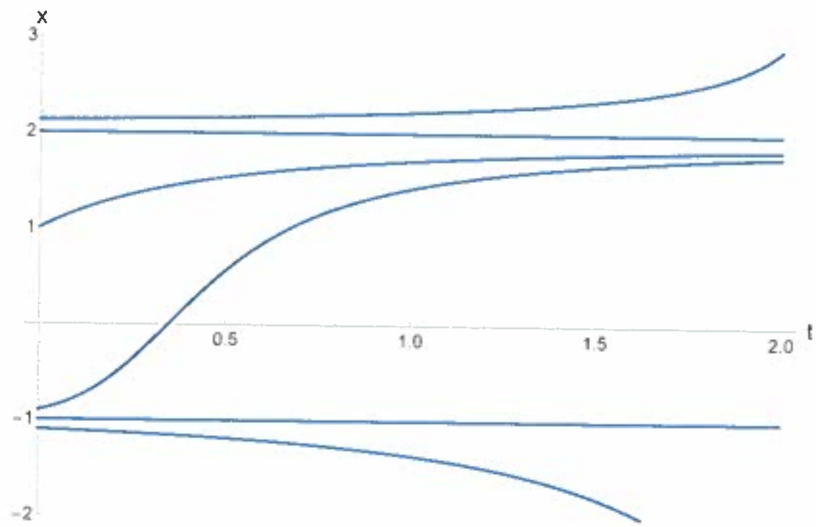


Figure 1:

(a) Sketch a graph of $f(x)$ that is consistent with the above figure.

(b) Give a formula for $f(x)$ that is consistent with the above figure.

4. For each of (a)-(d) find an equation $\frac{dx}{dt} = f(x)$ with the stated properties, or if there are no examples, explain why not. In each problem, assume that f is a smooth function, i.e. infinitely differentiable.

(a) Every real number is a fixed point.

(b) Every integer is a fixed point, and there are no others.

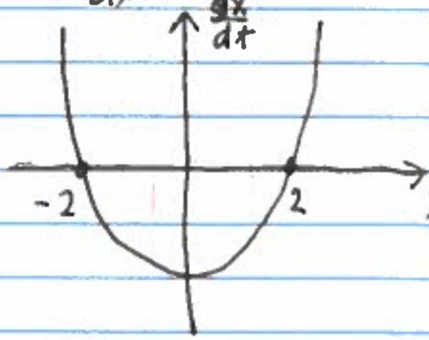
(c) There are no fixed points.

(d) There are precisely 100 fixed points.

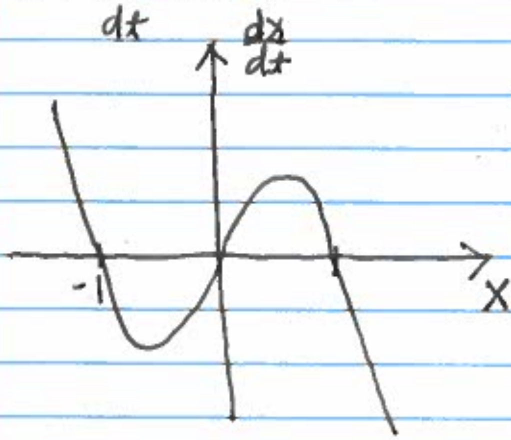
Homework #6

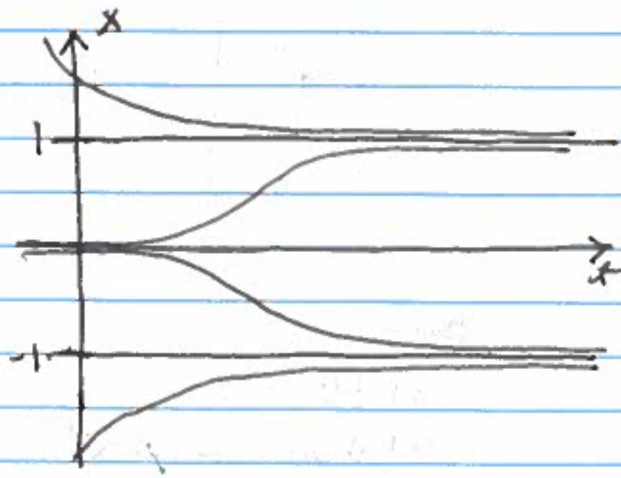
P.D.F. #1

a.) $\frac{dx}{dt} = 4x^2 - 16$



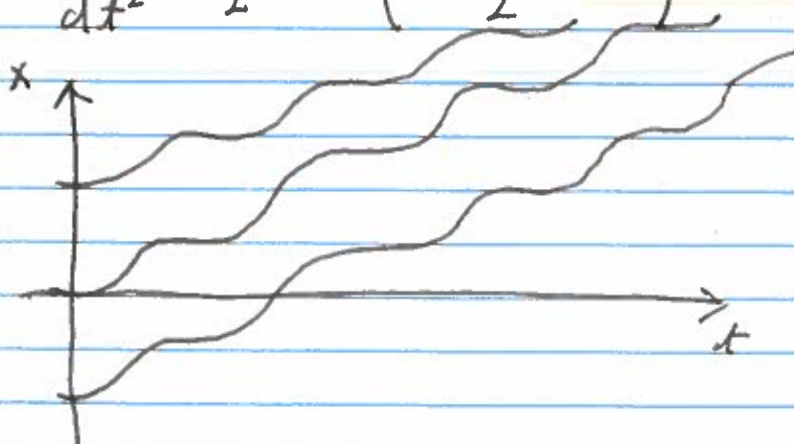
b.) $\frac{dx}{dt} = x - x^3$



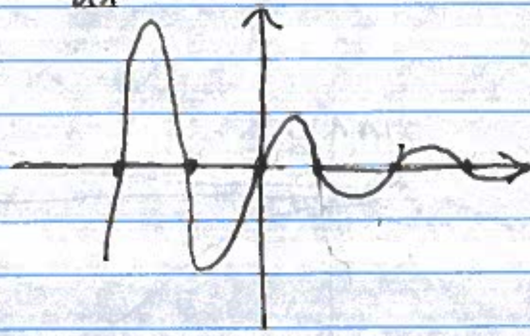


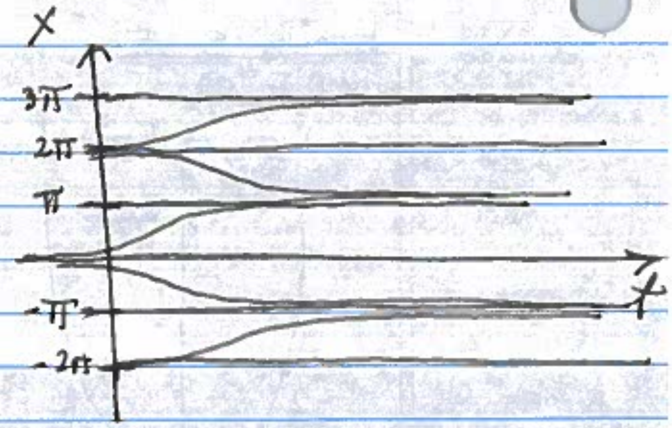
c.) $\frac{dx}{dt} = 1 + \frac{1}{2}\cos(x)$

$\Rightarrow \frac{d^2x}{dt^2} = -\frac{1}{2}\sin(x)\left(1 + \frac{1}{2}\cos(x)\right)$

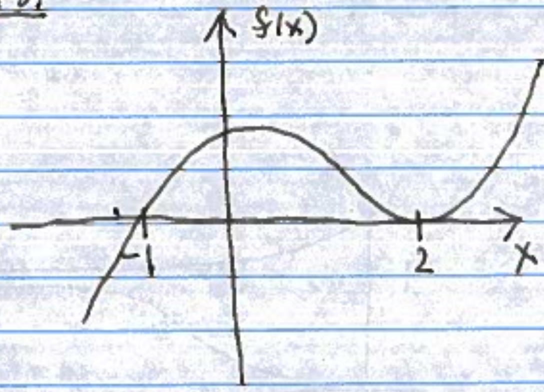


(c). $\frac{dx}{dt} = e^{-x}\sin(x)$





#3.



$f(x) = (x-2)^2(x+1)$

#4

(a) $f(x) = 0$

(b) $f(x) = \sin(\pi x)$

(c) $f(x) = 1$

(d) $f(x) = (x-1)\cdots(x-100)$

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$$\frac{dy}{dx} = \frac{x^2y^3+3xy^3}{2x^2y} = \frac{y^2(x+3)}{2x}$$

$$\Rightarrow \int \frac{1}{y^2}\,dy = \int\left(\frac{1}{2}+\frac{3}{x}\right)dx$$

$$\Rightarrow -\frac{1}{y} = \frac{1}{2}x + 3\ln(|x|) + C$$

$$\Rightarrow y = -\frac{1}{\frac{1}{2}x + \frac{3}{2}\ln(|x|) + C}$$

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$$3\frac{dy}{dx} = 2xy - y$$

$$y(2) = 1$$

$$\Rightarrow \frac{3}{y}\frac{dy}{dx} = 2x-1$$

$$\Rightarrow \int_1^y \frac{3}{y}\,dy = \int_2^x (2x-1)\,dx$$

$$\Rightarrow 3\ln(|y|) = x^2 - x - 4 + 2$$

$$\Rightarrow \ln(|y|) = \frac{x^2-x-2}{3}$$

$$\Rightarrow |y| = \exp\left(\frac{x^2-x-2}{3}\right)$$

To satisfy initial conditions

$$y(x) = \exp\left(\frac{x^2-x-2}{3}\right).$$