Section 3.1: Introduction to Differential Equations.

Newton's Law of Cooling:

hot coal in freezer.

$T(t)$ temperature of coal

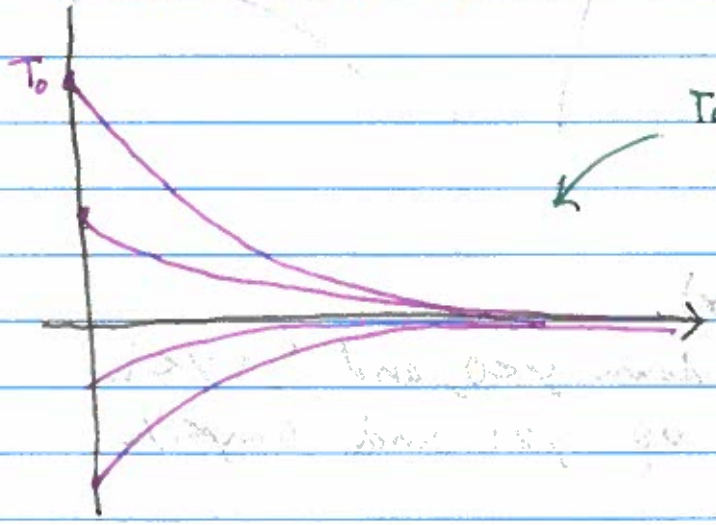
$$\frac{dT}{dt} = -KT, \quad T(0) = T_0$$

(K: constant.)

temperature change is proportional to current temperature.

Find $T(t)$

$$\Rightarrow T(t) = T_0 e^{-Kt}.$$



Example:

Solve

$$\frac{dy}{dx} = x^2$$

$$\Rightarrow y = \frac{1}{3}x^3 + C$$

general solution

Example:

Solve

$$\frac{dy}{dx} = x^3$$

$$y(1) = 1$$

} Initial value problem.

$$\Rightarrow y(x) = \frac{1}{4}x^4 + C$$

$$y(1) = 1/4 + C = 1$$

$$\Rightarrow C = 3/4$$

Example:

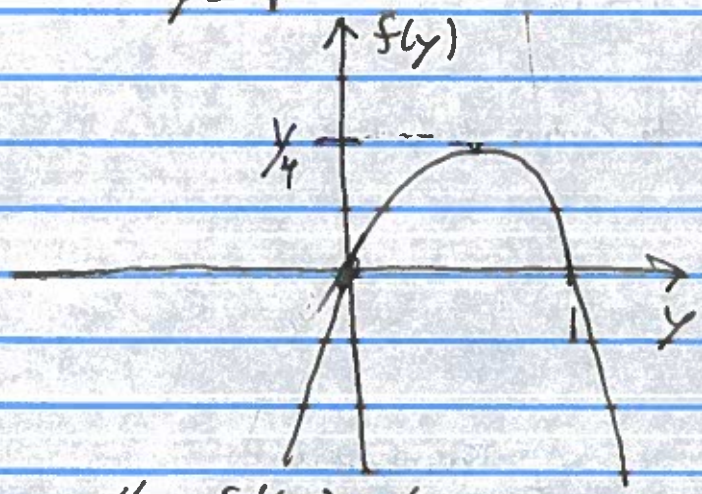
$y' = y(1-y) = f(y)$

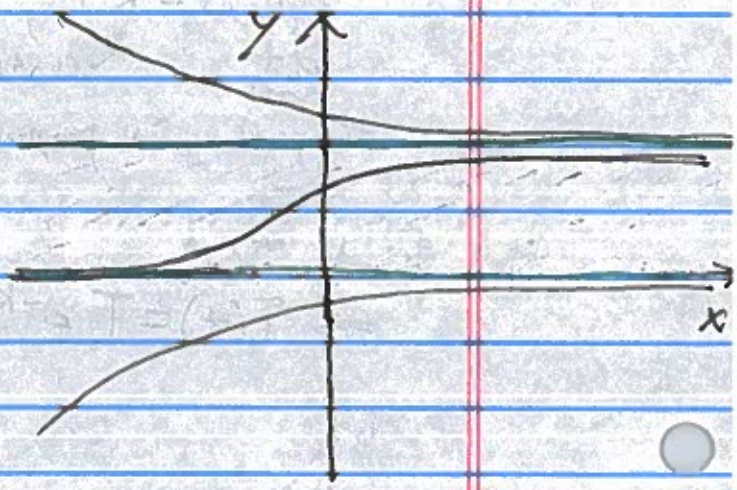
What do the solution curves look like??

Exact Solutions:

$y = 0$

$y = 1$





$y'' = f'(y)\,y'$

$= f'(y)\,f(y)$

$= (1-2y)\,y(1-y)$

Solution is concave down $y<0$, and $\frac{1}{2}<y<1$.

concave up $y>1$ and $0<y<\frac{1}{2}$