

Section 4.3: Method of Undetermined Coefficients

Example:

$$y'' - 3y' - 4y = 3e^{2t} \rightarrow \text{Find the general solution.}$$

$$\left(\frac{d^2}{dt^2} - 3\frac{d}{dt} - 4\right)y = 3e^{2t}$$

$$Ly = 3e^{2t}$$

Homogeneous solutions:

$$y_h = e^{\lambda t}$$

$$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda = 4, -1$$

$$y_h = c_1 e^{4t} + c_2 e^{-t}$$

Particular solution:

$$y_p = Ae^{2t}$$

$$\Rightarrow 4A - 6A - 4A = 3$$

$$A = -\frac{1}{2}$$

General Solution:

$$y = c_1 e^{4t} + c_2 e^{-t} - \frac{1}{2}e^{2t}$$

Example:

$$y'' - 3y' - 4y = -8e^x \cos(2x)$$

$$y_p = Ae^x \cos(2x) + Be^x \sin(2x)$$

$$y_p' = Ae^x \cos(2x) - 2Ae^x \sin(2x) + Be^x \sin(2x) + 2Be^x \cos(2x)$$

$$y_p'' = Ae^x \cos(2x) - 2Ae^x \sin(2x) - 2Ae^x \sin(2x) - 4Ae^x \cos(2x) + Be^x \sin(2x) + 2Be^x \cos(2x) + 2Be^x \cos(2x) - 4Be^x \sin(2x)$$

$$= (-3A + 4B) \cos(2x) + (-4A - 3B) \sin(2x) = -8e^x \cos(2x)$$

$$\begin{aligned} -3A + 4B &= -8 \\ -4A - 3B &= 0 \end{aligned} \Rightarrow \begin{bmatrix} -3 & 4 & -8 \\ -4 & -3 & 0 \end{bmatrix} \begin{array}{l} \\ -R_1 \end{array} \Rightarrow \begin{bmatrix} -3 & 4 & -8 \\ +1 & +7 & 0 \end{bmatrix} \begin{array}{l} +3R_1 \\ \\ \end{array}$$

$$\Rightarrow \begin{bmatrix} 0 & 11 & -8 \\ 1 & 7 & 0 \end{bmatrix}$$

Example:

$$y'' + 9y = t^2 e^{3t} + 6$$

Homogeneous Solution:

$$y = C_1 \cos(3t) + C_2 \sin(3t)$$

Particular Solution:

$$y_p = At^2 e^{3t} + Bte^{3t} + Ce^{3t} + D$$

$$y_p' = 2At e^{3t} + 3At^2 e^{3t} + B e^{3t} + 3Bte^{3t} + 3Ce^{3t}$$

$$y_p'' = 6At e^{3t} + 2Ae^{3t} + 6At e^{3t} + 9At^2 e^{3t} + 3B e^{3t} + 3B e^{3t} + 9Bte^{3t} + 9Ce^{3t}$$

$$t^2 e^{3t}: \quad 9A = 1$$

$$te^{3t}: \quad 12A + 9B + 9B = 0$$

$$e^{3t}: \quad 2A + 6B + 9C + 9C = 0$$

$$\frac{1}{9D} = 6$$

$$\left[\begin{array}{ccc|c} 9 & 0 & 0 & 1 \\ 12 & 12 & 0 & 0 \\ 2 & 6 & 18 & 0 \end{array} \right]$$

$$A = \frac{1}{9}$$

$$\rightarrow \frac{4}{3} + 12B = 0$$

$$B = -\frac{1}{9}$$

$$\Rightarrow \frac{2}{9} - \frac{6}{9} + 18C = 0$$

$$\Rightarrow -\frac{4}{9} = -18C$$

$$C = -$$

Example:

$$-1. \frac{d^2x}{dt^2} - \frac{dx}{dt} = e^x + x$$

$$3x^2 = x^2 + 2x$$

$$2x^2 = 2x$$

$$(x+1)(x-1) = 0$$

$$x = -1, 1$$

$$0 = x^2 - 2x + 1 = (x-1)^2$$

$$x^2 - 2x + 1 = (x-1)^2 = 0$$

$$x^2 - 2x + 1 = (x-1)^2 = 0$$

$$x^2 - 2x + 1 = (x-1)^2 = 0$$

$$x^2 - 2x + 1 = (x-1)^2 = 0$$

$$x^2 - 2x + 1 = (x-1)^2 = 0$$

$$x = 1$$

$$x = 1$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$x = 1$$

$$x = 1$$

$$x = 1$$

$$x = 1 + 18x = 18x$$

$$18x = 18x$$