Lecture 6: Linear Stability

Definitions

1. A linear system is spectrally stable if
$$Re(\lambda_i) < 0$$
for all λ_i.

2. The invariant subspaces are:
   a.) $E^u = span\{\vec{v}_j, \vec{w}_j : Re(\lambda_j) > 0\}$ (unstable subspace),
   b.) $E^c = span\{\vec{v}_j, \vec{w}_j ; Re(\lambda_j) = 0\}$ (center subspace),
   c.) $E^s = span\{\vec{v}_j, \vec{w}_j ; Re(\lambda_j) < 0\}$ (stable subspace),

   where
$$\vec{V}_j = \vec{v}_j + i\vec{w}_j$$
   are corresponding eigenvectors.
$$\mathbb{R}^n = E^u \oplus E^c \oplus E^s.$$

3. A linear system is hyperbolic if $Re(\lambda_i) \neq 0$ for all eigenvalues λ_i.

4. A linear system is linearly stable if all its solutions are bounded as $t \to \infty$.

5. A linear system is asymptotically stable if all all of it solutions satisfy $\lim_{t\to\infty} x(t) = 0$.

Linear Systems

asymptotically stable $\Leftrightarrow$ Spectrally stable $\Rightarrow$ linearly stable

Example:

$Re(\lambda_i) \leq 0$, does not imply linear stability

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{x} \Rightarrow \begin{array}{l} \dot{x} = y \\ \dot{y} = 0 \end{array} \Rightarrow t + x_0$$