Homework #4

#1. A multiple-choice exam consists of eight questions and three answers to each question. If a student guesses randomly, what is the probability that they will get exactly four correct answers?

Solution

Let X = # of correct answers. If we think of a correct answer as a success, then the probability is given by:

$$P(X=4) = p(4) = \binom{8}{4}\left(\tfrac{1}{3}\right)^4\left(\tfrac{2}{3}\right)^4$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} \frac{1}{3^4} \frac{2^4}{3^4}$$

$$= \frac{70 \cdot 2^4}{3^8}$$

$$= .1707$$

#2. A social scientist claims that only 15% of graduating English majors complete a novel in their lifetime. Assuming this claim is true, find the probabilities that among 18 graduating English majors

(a) exactly 10 will write a novel;

(b) at least 10 will write a novel;

(c) at most eight will write a novel.

Solution:

(a) Let X = # of English majors who write a novel.

Therefore,

$$P(X=10) = p(10) = \binom{18}{10}(.15)^{10}(.85)^{8} = 6.8758 \times 10^{-5}.$$

(b) $P(X \geq 10) = \sum_{x=10}^{18} \binom{18}{x} (.15)^x (.85)^{18-x} = 7.85695 \times 10^{-5}.$

(c) $P(X \leq 8) = \sum_{x=1}^{8} \binom{18}{x} (.15)^x (.85)^{18-x} = .945842.$ $\blacksquare$

#3.

If the probability is .75 that a person will believe a rumor, find the probabilities that

(a) the eighth person to hear the rumor will be the fifth to believe it.

(b) the fifteenth person to hear the rumor will be the tenth to believe it.

Solution:

(a) If we think of belief as a success then we must have four successes in 7 trials followed by another success

$$\Rightarrow p = \binom{7}{4} (.75)^4 \cdot (.25)^3 \cdot (.75) = \binom{7}{4} (.75)^5 \cdot (.25)^3 = .129776$$

(b) A similar calculation yields

$$p = \binom{14}{9} (.75)^9 (.25)^5 (.75) = \binom{14}{9} (.75)^{10} (.25)^5 = .110097$$

#4

Among the 16 applicants for a job, 10 have college degrees. If three of the applicants are randomly chosen for interviews, what are the probabilities that

(a) none have a college degree;

(b) one has a college degree;

(c) two have college degrees;

(d) all three have college degrees

Solution:

Let X = # that have college degrees.

(a) $P(X=0) = \frac{6}{16} \cdot \frac{5}{15} \cdot \frac{4}{14} = .0357$

(b) $P(X=1) = \binom{3}{1} \frac{10}{16} \cdot \frac{6}{15} \cdot \frac{5}{14} = .2679$

(c) $P(X=2) = \binom{3}{2} \frac{10}{16} \cdot \frac{9}{15} \cdot \frac{6}{14} = .4821$

(d) $P(X=3) = \frac{10}{16} \cdot \frac{9}{15} \cdot \frac{8}{14} = .2143$

#5

If six letters are randomly selected from the word Mississippi, what is the probability that the letters could be arranged to spell the word "simpis"

Solution:

The probability of exactly forming the word simpis is

$p = \frac{4}{11} \cdot \frac{4}{10} \cdot \frac{1}{9} \cdot \frac{2}{8} \cdot \frac{3}{7} \cdot \frac{3}{6}.$

Therefore, accounting for all of the arrangements of simpis we have that

$$p = \frac{(4)^2 \cdot 2 \cdot (3)^2}{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6} \cdot \frac{6!}{2!\,2!} = .1558$$