MTH 357/657
Homework #6

Due Date: March 03, 2023

1 Moment Generating Functions

1. Given that X has the probability distribution $f(x) = \frac{1}{8}\binom{3}{x}$ for $x = 0, 1, 2,$ and 3, find the moment-generating function of this random variable and use it to determine μ_1' and μ_2'.

2. Find μ, μ_2', and σ^2 for the random variable X that has the probability distribution $f(x) = 1/2$ for $x = -2$ and $x = 2$.

3. If the random variable X has the mean μ and the standard deviation σ, show that the random variable
$$Z = \frac{X - \mu}{\sigma}$$
satisfies
$$\mathbb{E}(Z) = 0 \text{ and } \mathbb{E}\left(Z^2\right) = 1.$$

4. The symmetry or skewness (lack of symmetry) of a distribution is often measured by means of the quantity
$$\alpha_3 = \frac{\mu_3}{\sigma^3}.$$
Draw histograms and calculate α_3 for probability distributions $f(x)$ and $g(x)$ satisfying
   (a) $f(1) = .05$, $f(2) = .15$, $f(3) = .30$, $f(4) = .30$, $f(5) = .15$, and $f(6) = .05$;
   (b) $g(1) = .05$, $g(2) = .20$, $g(3) = .15$, $g(4) = .45$, $g(5) = .10$, and $g(6) = .05$.

   The first distribution is symmetrical while the second has a "tail" on the left-hand side and is said to be negatively skewed.

5. The extent to which a distribution is peaked or flat, also called the kurtosis of the distribution, is often measured by means of the quantity
$$\alpha_4 = \frac{\mu_4}{\sigma^4}.$$
Draw histograms and calculate α_4 for probability distributions $f(x)$ and $g(x)$ satisfying
   (a) $f(-3) = .06$, $f(-2) = .09$, $f(-1) = .10$, $f(0) = .5$, $f(1) = .10$, $f(2) = .09$, and $f(3) = .06$.
   (b) $f(-3) = .04$, $f(-2) - .11$, $f(-1) = .20$, $f(0) = .30$, $f(1) = .20$, $f(2) = .11$, and $f(3) = .04$.

6. Find the moment generating function of the discrete random variable X that has the probability distribution
$$f(x) = 2\left(\frac{1}{3}\right)^x \text{ for } x = 1, 2, 3, \ldots$$
and use it to determine the values of μ_1' and μ_2'.

2 Tchebysheff's Theorem

1. What is the smallest value of k in Tchebysheff's theorem for which the probability that random variable will take on a value between $\mu - k\sigma$ and $\mu + k\sigma$ is

   (a) at least .95;

   (b) at least .99.

2. If we let $k\sigma = c$ in Tchebysheff's theorem, what does this theorem assert about the probability that a random variable will take on a value between $\mu - c$ and $\mu + c$.

3. The number of marriage licenses issued in a certain city during the month of June may be looked upon as a random variable with $\mu = 124$ and $\sigma = 7.5$. According to Tchebysheff's theorem, with what probability can we assert that between 64 and 184 marriage licenses will be issued during the month of June.