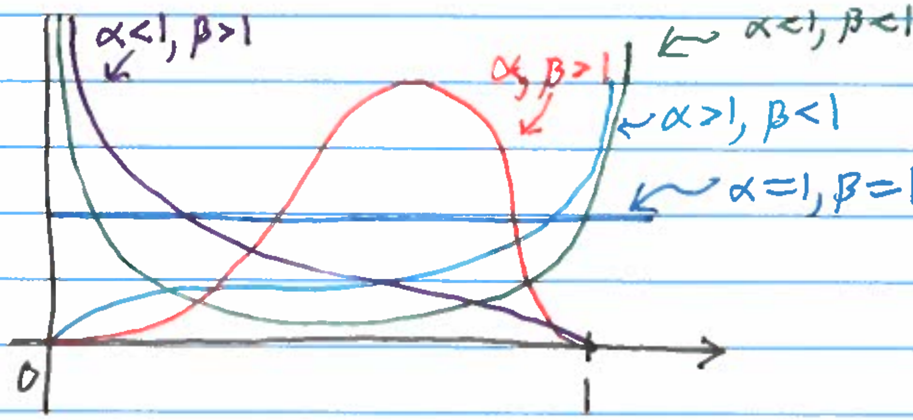
Lecture 21: The Beta Distribution

Definition - A random variable X has a beta distribution if its density is given by

$$p(x) = \begin{cases} K x^{\alpha-1}(1-x)^{\beta-1}, & 0 \le y \le 1, \quad (\alpha, \beta > 0). \\ 0 & \end{cases}$$

$$K = \frac{1}{\beta(\alpha,\beta)}, \quad \beta(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$



Properties:

$$\mathbb{E}[X] = \frac{1}{\beta(\alpha,\beta)} \int_0^1 x^{\alpha}(1-x)^{\beta-1}\,dx$$

$$= \frac{1}{\beta(\alpha,\beta)} \cdot \frac{\beta(\alpha+1,\beta)}{\beta(\alpha+1,\beta)} \int_0^1 x^{\alpha}(1-x)^{\beta-1}\,dx$$

$$= \frac{\beta(\alpha+1,\beta)}{\beta(\alpha,\beta)}$$

$$= \frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+\beta+1)} \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$= \frac{\alpha}{\alpha+\beta}$$