Lecture 4: Probability and Poker

Example

In a standard hand of poker what is the prob. of drawing

- one pair, E_1
- two pair, E_2
- 3 of a kind, E_3
- full house, E_4
- four of a kind E_5

13 numeric kinds
4 of each kind

$S = \{$five card hands$\}$
$= \{ABCDE :$ each letter $\in \{\#, S\}$ where $\# \in \{1, .., 10, J, Q, K, A\}$
$S \in \{\heartsuit, \spadesuit, \diamondsuit, \clubsuit\}$ and order does not matter$\}$.

$$|S| = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = \binom{52}{5} = 2{,}598{,}960$$

pair:

$$P(E_1) = \underbrace{\binom{13}{1} \cdot \binom{4}{2}}_{\text{\# of pairs}} \underbrace{\binom{12}{3} \binom{4}{1}^3}_{\text{remaining possible cards}} \Big/ \binom{52}{5}$$

$$= 13 \cdot \binom{4}{2} \frac{12 \cdot 11 \cdot 10}{3!} \cdot \binom{4}{1}^3 \Big/ \binom{52}{5}$$

$$= \frac{13 \cdot 3 \cdot 2 \cdot 12 \cdot 11 \cdot 10 \cdot 4^3}{2{,}598{,}960}$$

$$= .4225$$

two pair:

AABBC

$$P(E_2)=\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}\Big/\binom{52}{5}=\frac{13\cdot 12}{2}\left(\frac{4\cdot 3}{2}\right)^2\cdot 11\cdot 4\Big/\binom{52}{5}$$

$$\Rightarrow P(E_2)=13\cdot 36\cdot 6\cdot 11\cdot 4\Big/\binom{52}{5}=.047539$$

three pair:

$$P(E_3)=\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2\Big/\binom{52}{5}$$

$$=13\cdot 4\cdot 6\cdot 11\cdot 4^2/\binom{52}{5}$$

$$=.0211$$

full house:

$$P(E_4)=\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}\Big/\binom{52}{5}$$

$$=13\cdot 4\cdot 12\cdot 6/\binom{52}{5}$$

$$=.0014$$

Four of a kind:

$$P(E_5)=\binom{13}{1}\cdot\binom{12}{1}\cdot\binom{4}{1}\Big/\binom{52}{5}$$

$$=13\cdot 12\cdot 4/\binom{52}{5}$$

$$=.000240$$