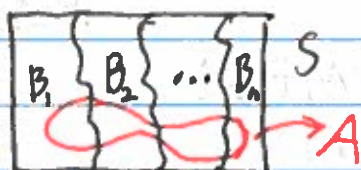


Lecture 7: Bayes' Theorem

Suppose S can be partitioned into sets



$$\Rightarrow P(A) = P((A \cap B_1) \cup (A \cap B_2) \cup \dots \cup (A \cap B_n))$$

$$P(A) = P(A \cap B_1) + \dots + P(A \cap B_n)$$

$$P(A) = P(A|B_1)P(B_1) + \dots + P(A|B_n)P(B_n)$$

total law of probability.

Example:

.60 = prob. of a strike

.85 = prob of completing job if no strike

.35 = prob. of completing job if there is a strike

What is the probability the job will be completed.

A = job will be completed

B = there is a strike

$$\Rightarrow P(A) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$$

$$= .35 \cdot .60 + .85(1 - .60)$$

$$= .55$$

Suppose B_1, \dots, B_n partition S .

$$P(A|B_r) = \frac{P(A \cap B_r)}{P(B_r)}, \quad P(B_r|A) = \frac{P(A \cap B_r)}{P(A)}$$

$$\Rightarrow P(A|B_r)P(B_r) = P(B_r|A)P(A)$$

$$\Rightarrow \frac{P(A|B_r)P(B_r)}{P(A)} = P(B_r|A)$$

$$\Rightarrow \boxed{P(B_r|A) = \frac{P(A|B_r)P(B_r)}{P(A|B_1) + \dots + P(A|B_n)}}$$

Bayes' Theorem.

Example:

Consulting firm rents cars from three agencies:

- 60% from agency 1

- 30% from agency 2

- 10% from agency 3

Q1:

If 9% from A1 need tune up, 20% from A2 need tune up, 6% from agency A3 need tune up. What is probability a car delivered to the agency needs a tune up?

$$P(T) = .6 \cdot .09 + .30 \cdot .20 + .10 \cdot .06 = .12$$

Q2:

If the car needs a tune up, what is the probability it came from agency 2?

$$P(2|T) = \frac{P(2) \cdot P(T|2)}{P(T|1) + P(T|2) + P(T|3)} = \frac{.30 \cdot .20}{.12} = .5$$