Lecture 8: Random Variables and Distributions

Definition: If S is a sample space and $X: S \to \mathbb{R}$ is a function, then X is called a **random variable**.

Example:

Roll two six sided dice and let X = sum of both dice.

- $X = x \to$ is the set in S such that $X(y) = x$ for all $y \in X = x$

- $X = 9 = \{(3,6), (4,5), (5,4), (6,3)\}$
- $P(X=9) = \frac{4}{36} = \frac{1}{12}$.

Example:

Drawer contains 5 brown socks and 3 green socks. Two socks are selected at random.

$S = \{(BB), (BG), (GB), (GG)\}$

W = number of brown socks

$W(BB) = 2, W(BG) = 1, W(GB) = 1, W(GG) = 0$

$W=2 = \{BB\}, W=1 = \{(BG), (GB)\}, W=0 = \{GG\}$

a.) $P(W=2) = P(\{BB\}) = \frac{5}{8} \cdot \frac{4}{7} = \frac{5}{14}$

b.) $P(W=1) = P(\{(BG), (GB)\}) = P(\{BG\}) + P(\{GB\})$

$\Rightarrow P(W=1) = \frac{5}{8} \cdot \frac{3}{7} + \frac{3}{8} \cdot \frac{5}{7} = \frac{30}{56} = \frac{15}{28}$

c) $P(W=0) = P(\{GG\}) = \frac{3}{8} \cdot \frac{2}{7} = \frac{6}{56}$.

Probability Distribution for a discrete random variable is the function

$$p(x) = P(X = x)$$

Example:

Returning to the sock example

$p(0) = 6/56 = .107 =$

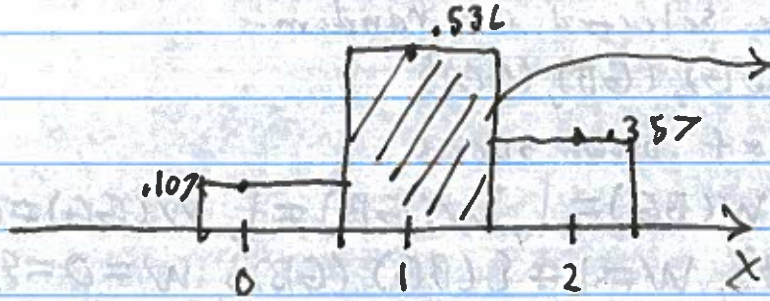
$p(1) = 15/28 = .536$

$p(2) = 5/14 = .357$

We can also find a formula

$$p(0) = \frac{\binom{5}{2}\binom{3}{0}}{\binom{8}{2}}, \quad p(1) = \frac{\binom{5}{1}\binom{3}{1}}{\binom{8}{2}}, \quad p(2) = \frac{\binom{5}{0}\binom{3}{2}}{\binom{8}{2}}$$

$$\Rightarrow p(x) = \binom{5}{2-x}\binom{3}{x} \Big/ \binom{8}{2}$$



The area in each bar is equal to the probability $P(X = x) = p(x)$.

Properties:

1. $0 \le p(x) \le 1$, for all x
2. $\sum_x p(x) = 1$.