

MTH 357/657

Quiz #6

1. Suppose X is a discrete random variable with probability distribution function satisfying

$$f(-2) = \frac{1}{4}, f(-1) = \frac{1}{3}, f(1) = \frac{1}{4} \text{ and } f(2) = \frac{1}{6}$$

on the domain $x = -2, -1, 1, 2$. Find the moment generating function of this random variable and use it find μ'_1, μ'_2 .

$$m(t) = \mathbb{E}[e^{tX}] = \frac{e^{-2t}}{4} + \frac{e^{-t}}{3} + \frac{e^t}{4} + \frac{e^{2t}}{6}$$

$$\Rightarrow m'(t) = -\frac{e^{-2t}}{2} - \frac{e^{-t}}{3} + \frac{e^t}{4} + \frac{2e^{2t}}{6}$$

$$\Rightarrow \mu'_1 = m'(0) = -\frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{3} = -\frac{1}{4}$$

$$\mu'_2 = m''(0) = \left. e^{-2t} + \frac{e^{-t}}{3} + \frac{e^t}{4} + \frac{4e^{2t}}{6} \right|_0 = 1 + \frac{1}{3} + \frac{1}{4} + \frac{2}{3} = \frac{9}{4}$$