MTH 357/657
Quiz #8

1. Suppose X is a continuous random variable with probability density function

$$p(x) = \begin{cases} 2xe^{-x^2} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Find the exact value of $\mathbb{E}[X]$. You can use the fact that $\Gamma(1/2) = \sqrt{\pi}$.

$$\mathbb{E}[X] = \int_0^\infty 2x^2 e^{-x^2} dx \qquad \left(u = x^2, \ du = 2x\, dx \right.$$
$$\left. x = u^{1/2}, \ dx = du/2x \right)$$

$$= \int_0^\infty u^{1/2} e^{-u}\, du$$

$$= \Gamma(3/2)$$

$$= \tfrac{1}{2} \Gamma(1/2) = \frac{\sqrt{\pi}}{2}$$

(b) Find the exact value of σ^2 for this random variable.

$$\mathbb{E}[X^2] = \int_0^\infty 2x^3 e^{-x^2} dx \qquad \left(u = x^2, \ du = 2x\, dx \right.$$
$$\left. x = u^{1/2}, \ dx = du/2x \right)$$

$$= \int_0^\infty u\, e^{-u}\, du$$

$$= \Gamma(2)$$

$$= 1$$

Therefore,

$$\sigma^2 = 1 - \frac{\pi}{4}.$$