1. Suppose X, Y are continuous random variables with joint probability density function
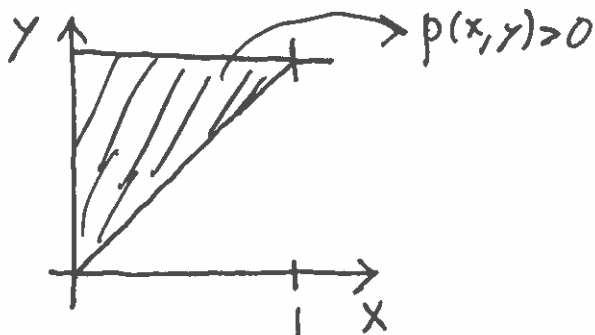
$$p(x,y) = \begin{cases} k(1-y) & 0 < x < y, \ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases},$$

   where $k > 0$ is a constant.

   (a) Find the value of k.

   (b) Find the marginal density for X.

   (c) Find the conditional density for Y given $X = x$.

(a) $1 = \int_0^1 \int_x^1 k(1-y)\,dy\,dx$

$= k \int_0^1 \left(y - y^2/2\right)\Big|_x^1 dx$

$= k \int_0^1 \left(1 - \tfrac{1}{2} - x + \tfrac{x^2}{2}\right) dx$

$= k\left(\tfrac{1}{2} + \left(-\tfrac{x^2}{2} + \tfrac{x^3}{6}\right)\Big|_0^1\right)$

$= k\left(\tfrac{1}{2} - \tfrac{1}{2} + \tfrac{1}{6}\right)$

$\Rightarrow k = 6.$

b. From the previous calculation

$f(x) = \begin{cases} 6\left(\tfrac{1}{2} - x + \tfrac{x^2}{2}\right), & 0 < x < 1 \\ 0 & , \text{elsewhere} \end{cases}$

c). $p(y|x) = \begin{cases} \dfrac{(1-y)}{\tfrac{1}{2} - x + \tfrac{x^2}{2}}, & 0 < x < y, \ 0 < y \\ 0 & , \text{elsewhere} \end{cases}$