MTH 225
Quiz #6

1. Suppose $A \in M_{2\times 2}(\mathbb{C})$ is a Hermitian matrix with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = -3$ with corresponding eigenvectors

$$\vec{v}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \text{ and } \vec{v}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

(a) Compute $A\vec{v}_1$ and $A\vec{v}_2$.

$$A\vec{v}_1 = 2\vec{v}_1 = \frac{2}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}, \quad A\vec{v}_2 = \frac{-3}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

(b) Find the diagonlization of A.

$$A = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & -3\end{bmatrix}\begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}$$

(c) Find the SVD of A.

$\sigma_1 = 3$

$\sigma_2 = 2$

$A\vec{v}_2 = \sigma_1 \vec{u}_1$

$\Rightarrow \vec{u}_1 = \dfrac{A\vec{v}_2}{3} = -1/\sqrt{2}\begin{bmatrix}1\\-1\end{bmatrix}$

$A\vec{v}_1 = \sigma_2 \vec{u}_2$

$\Rightarrow \vec{u}_2 = \dfrac{A\vec{v}_1}{2} = 1/\sqrt{2}\begin{bmatrix}1\\1\end{bmatrix}$

$$\Rightarrow A = U\Sigma V^*$$

$$= \begin{bmatrix}-1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{bmatrix}$$