# MTH 352/652: Homework #1

Due Date: January 26, 2024

## 1 Problems for Everyone

1. Sign up for Piazza. I will check the roster for your name.

2. Determine which of the following operators are linear. If an operator is linear, prove it. If an operator is nonlinear find a counterexample, i.e., two functions $u, v$ for which $L[u+v] \neq L[u] + L[v]$ or a function $u$ for which $L[cu] \neq cL[u]$.

   (a) $L[u] = u_x + xu_t$
   (b) $L[u] = u_x + uu_t$
   (c) $L[u] = u_x + u_t^2$
   (d) $L[u] = u_x + u_t + 1$
   (e) $L[u] = \sqrt{1+x^2}\cos(t)u_x + u_{txt} - \arctan(x/y)u$

3. For each of the following equations, state the order and whether it is nonlinear, linear homogenous, or linear inhomogeneous. There is no need prove anything for this problem.

   (a) $u_t - u_{xx} + 1 = 0$
   (b) $u_t - u_{xx} + xu = 0$
   (c) $u_t - u_{xxt} + uu_x = 0$
   (d) $u_{tt} - u_{xx} + x^2 = 0$
   (e) $u_x(1+u_x^2)^{-1/2} + u_t(1+u_t^2)^{-1/2} = 0$
   (f) $u_x + e^t u_t = 0$
   (g) $u_t + u_{xxxx} + \sqrt{1+u} = 0$

4. Verify that $u(x,y) = f(x)g(y)$ is a solution to the PDE $uu_{xy} = u_x u_y$ for all pairs of differentiable functions $f$ and $g$ of one variable.

5. Show that the following functions solve the PDE $u_{xx} + u_{yy} = 0$:

   (a) $u(x,y) = e^x \cos(y)$
   (b) $u(x,y) = 1 + x^2 - y^2$
   (c) $u(x,y) = x^3 - 3xy^2$
   (d) $u(x,y) = \ln(x^2+y^2)$
   (e) $u(x,y) = \arctan(y/x)$
   (f) $u(x,y) = \dfrac{x}{x^2+y^2}$
   (g) $u(x,y) = \sin(nx)(e^{ny} - e^{-ny})$, $n \in \mathbb{R}$

6. Solve the PDE $3u_t + u_{xt} = 0$. **Hint:** Let $v = u_t$.

7. Find the general solution to the PDE $u_{xy} = 0$ in terms of two arbitrary functions.

8. Find a function $u(t, x)$ that satisfies the PDE

$$u_{xx} = 0$$

on the domain $0 < x < 1$, $t > 0$ subject to the boundary conditions $u(t, 0) = t^2$ and $u(t, 1) = 1$.

9. Show that the nonlinear equation $u_t = u_x^2 + u_{xx}$ can be reduced to the linear equation $w_t = w_{xx}$ by changing the variable to $w = e^u$.

10. Solve the following initial value problems and graph the solutions at times $t = 1, 2$ and 3.

    (a) $u_t - 3u_x = 0$ with $u(0, x) = e^{-x^2}$

    (b) $2u_t + 3u_x = 0$ with $u(0, x) = \sin(x)$

    (c) $u_t + 2u_x = 0$ with $u(-1, x) = x/(1 + x^2)$

    (d) $u_t + u_x + u = 0$ with $u(0, x) = \arctan(x)$