MTH 352/652: Homework #10

Due Date: April 26, 2024

1 Problems for Everyone

1. pg. #283, #7.3.10, #7.3.11, #7.3.12, #7.3.14.
2. Find the convolution of the functions $f(x) = x$ and $g(x) = e^{-x^2}$.
3. Consider the following initial value problem for the heat equation with proportional heat loss:

$$\begin{aligned} u_t &= Du_{xx} - au, \ x \in \mathbb{R}, \ t > 0, \\ u(0, x) &= e^{-x^2}, \end{aligned}$$

   where $D > 0$ and $a > 0$ are constants. Using Fourier transforms find a formula for the solution to this initial value problem.

4. Consider the following initial value problem for the heat equation with advection:

$$\begin{aligned} u_t &= Du_{xx} - cu_x, \ x \in \mathbb{R}, \ t > 0, \\ u(0, x) &= e^{-x^2}, \end{aligned}$$

   where $D > 0$ and $c > 0$ are constants. Using Fourier transforms find a formula for the solution to this initial value problem.

5. Use Fourier transforms to find bounded solutions to the following differential equation on $\mathbb{R}$:

$$-u''(x) + u(x) = e^{-|x|}.$$

6. Consider the following initial value problem for the heat equation:

$$\begin{aligned} u_t &= Du_{xx}, \ x \in \mathbb{R}, \ t > 0, \\ u(0, x) &= f(x), \end{aligned}$$

   where $D > 0$ is a constant. Show that if $f(x)$ is an odd function then $u(t, x)$ is an odd function in x.