MTH 352/652: Homework #11

Due Date: May 01, 2024

1 Problems for Everyone

1. Consider the following sequence of functions

$$\delta_n(x) = \begin{cases} n - n^2|x| & |x| < 1/n \\ 0 & \text{otherwise.} \end{cases}$$

   (a) On the same set of axes, sketch $\delta_1(x)$, $\delta_2(x)$, and $\delta_4(x)$.

   (b) Explain why

$$\lim_{n \to \infty} \delta_n(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}.$$

   (c) Prove that if $f \in C^2$ then

$$\lim_{n \to \infty} \int_{-\infty}^{\infty} \delta_n(x) f(x) dx = f(0).$$

   (d) Explain why

$$\lim_{n \to \infty} n^{-1/2} \delta_n(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases}.$$

   and prove that if $f \in C^2$ then

$$\lim_{n \to \infty} \int_{-\infty}^{\infty} n^{-1/2} \delta_n(x) f(x) dx = 0.$$

2. Evaluate the following integrals

   (a) $\int_{-\pi}^{\pi} \delta(x) \cos(x) dx.$

   (b) $\int_{1}^{2} \delta(x)(x - 2) dx.$

   (c) $\int_{0}^{1} \delta\left(x - \frac{1}{3}\right) x^2 dx.$

   (d) $\int_{-1}^{1} \frac{\delta(x + 2)}{1 + x^2} dx.$

3. pg. 227, #6.1.4(a,c,d), #6.1.5(a,c).

4. pg. 240, #6.2.1.