1. Find a function $u(t, x)$ that satisfies the PDE

$$u_x = u$$

subject to the boundary condition $u(t, 1) = 1$.

$$\frac{\partial u}{\partial x} = u$$

$$\Rightarrow \ln(|u|) = x + g(t)$$
$$\Rightarrow u = e^{g(t)} e^x$$
$$\Rightarrow u = g(t) e^x$$

Therefore,
$$u(t, 1) = g(t) e = 1$$
$$\Rightarrow g(t) = \frac{1}{e}$$
$$\Rightarrow u(t, x) = e^{x-1}.$$