

MTH 352/652

Quiz #2

1. Find a function $u(t, x)$ that satisfies the PDE

$$u_t - tu_x = -u.$$

subject to the initial condition $u(0, x) = f(x)$.

Characteristic curves satisfy

$$\frac{dx}{dt} = -t$$

$$\Rightarrow x = -\frac{t^2}{2} + c$$

$$\text{Let } z = x + \frac{t^2}{2}, \tau = t$$

$$\Rightarrow \frac{\partial u}{\partial \tau} = -u$$

$$\Rightarrow u(\tau, z) = g(z)e^{-\tau}$$

$$\Rightarrow u(t, x) = g\left(x + \frac{t^2}{2}\right)e^{-t}$$

$$u(0, x) = g(x) = f(x)$$

$$\Rightarrow u(t, x) = f\left(x + \frac{t^2}{2}\right)e^{-t}.$$