MTH 352/652
Quiz #4

1. Find all separable solutions to the following partial differential equation

$$u_t = txu_x.$$

Let $u = T \cdot X$

$\Rightarrow u_t = T'X$

$u_x = TX'$

$\Rightarrow T'X = txTX'$

$\Rightarrow \frac{T'}{tT} = \frac{xX'}{X} = \lambda$

$\Rightarrow \frac{T'}{T} = \lambda t, \frac{X'}{X} = \frac{\lambda}{x}$

$\Rightarrow \ln(T) = \frac{\lambda}{2}t^2, \ln(X) = \lambda \ln(x) + c$

$\Rightarrow T = e^{\frac{\lambda}{2}t^2}, X = ce^{\lambda \ln(x)}$

$\Rightarrow T = e^{\frac{\lambda}{2}t^2}, X = ce^{\ln(x^\lambda)}$

$\Rightarrow T = e^{\frac{\lambda}{2}t^2}, X = cx^\lambda$

$\Rightarrow \boxed{u(t,x) = cx^\lambda e^{\frac{\lambda}{2}t^2}}$