MTH 352/652
Quiz #5

1. Let $f(x)$ be defined on the interval $[-1, 1]$ by

$$f(x) = \begin{cases} 0 & -1 \le x \le -\frac{1}{2} \\ x & -\frac{1}{2} < x < \frac{1}{2} \\ 0 & \frac{1}{2} \le x \le 1 \end{cases}.$$

(a) **Short Answer:** Is $f(x)$ an even or odd function?

Odd

(b) **Short Answer:** Given that $\{1, \cos(x), \sin(x), \cos(2x), \sin(2x), \ldots\}$ is an orthogonal system on $[-\pi, \pi]$, what is the the corresponding orthogonal system on the interval $[-1, 1]$?

$\{1, \cos(\pi x), \sin(\pi x), \cos(2\pi x), \sin(2\pi x), \ldots\}$

(c) Find the Fourier series of $f(x)$ on this $[-1, 1]$ and sketch the function that the Fourier series converges to on the interval $[-2, 2]$. **Be sure to carefully label your graph including any points of discontinuity.**

$f(x) = \sum_{n=1}^{\infty} b_n \sin(n\pi x)$

$\Rightarrow \int_{-1}^{1} f(x)\sin(n\pi x)\,dx = \int_{-1}^{1} b_n \sin^2(n\pi x)\,dx$

$= \int_{-1}^{1} b_n \frac{(1-\cos(2n\pi x))}{2}\,dx$

$= b_n$

$\Rightarrow b_n = \int_{-1}^{1} f(x)\sin(n\pi x)\,dx$

$= \int_{-1/2}^{1/2} x\sin(n\pi x)\,dx$

$= 2\int_{0}^{1/2} x\sin(n\pi x)\,dx$

$= -\frac{2x\cos(n\pi x)}{n\pi}\Big|_0^{1/2} + \frac{2}{n\pi}\int_0^{1/2} \cos(n\pi x)\,dx$

$= -\frac{\cos(\frac{n\pi}{2})}{n\pi} + \frac{2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$

$\Rightarrow f(x) = \sum_{n=1}^{\infty}\left(\frac{-\cos(\frac{n\pi}{2})}{n\pi} + \frac{2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right)\sin(n\pi x)$