

Exercises from Day 3 talk by Jeremy Rouse

6. Let Q be a positive-definite quaternary form with $D(Q)$ a fundamental discriminant. Factor $\chi_{D(Q)} = \prod_{p|2D(Q)} \chi_p$ as a product of Dirichlet characters with prime power moduli. Let $\epsilon_p(Q)$ be the Hasse invariant of Q/\mathbb{Q}_p .

(a) Show that if $p|2D(Q)$ is an odd prime and m is a positive integer coprime to p represented by Q^* , then $\chi_p(m) = \epsilon_p(Q)$.

(b) Show that if m is an odd integer represented by Q^* , then $\chi_2(m) = -\epsilon_2(Q)$.

(c) Conclude that if m is represented by Q^* , then either $\chi_D(m) = 0$ or $\chi_D(m) = -1$.