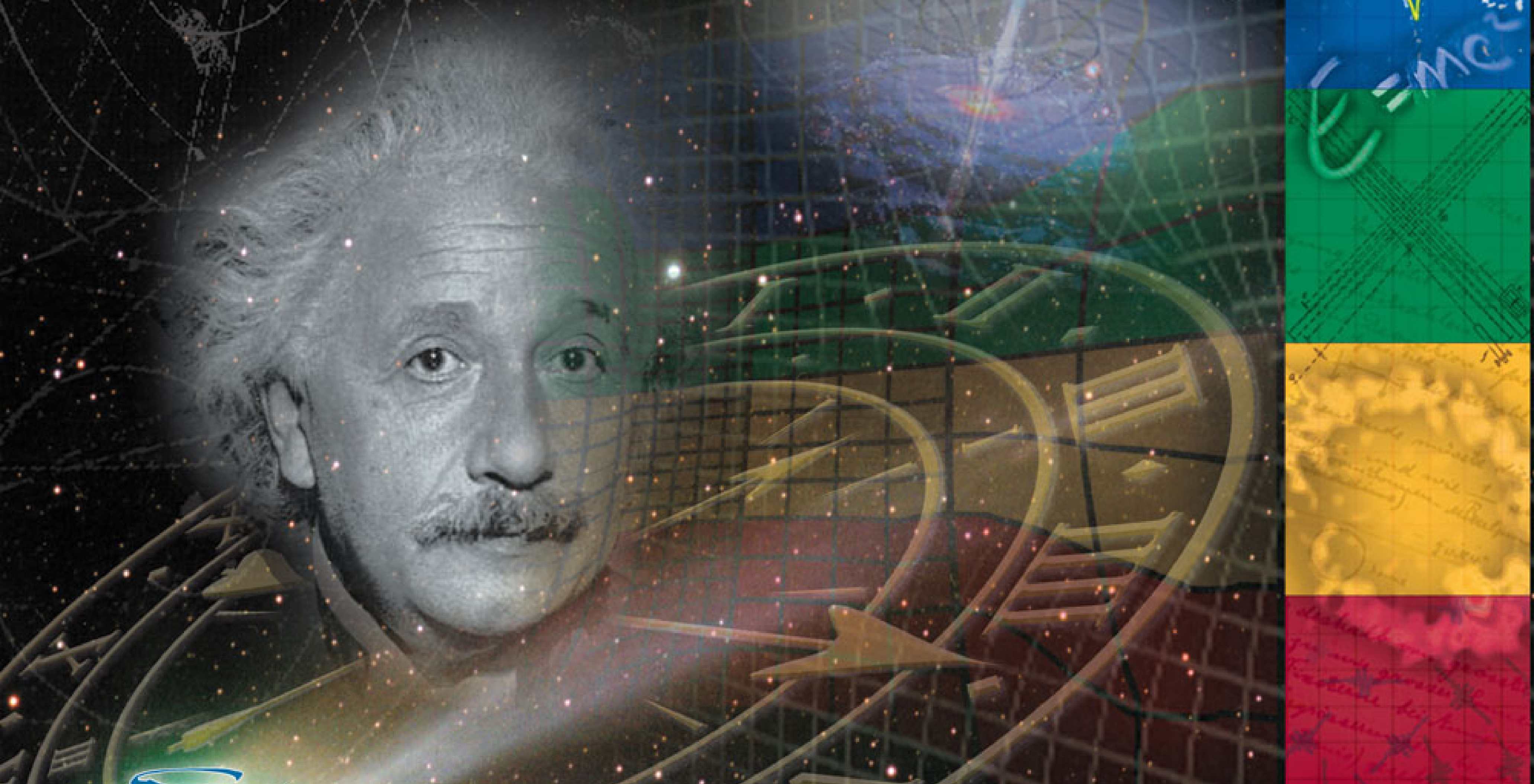
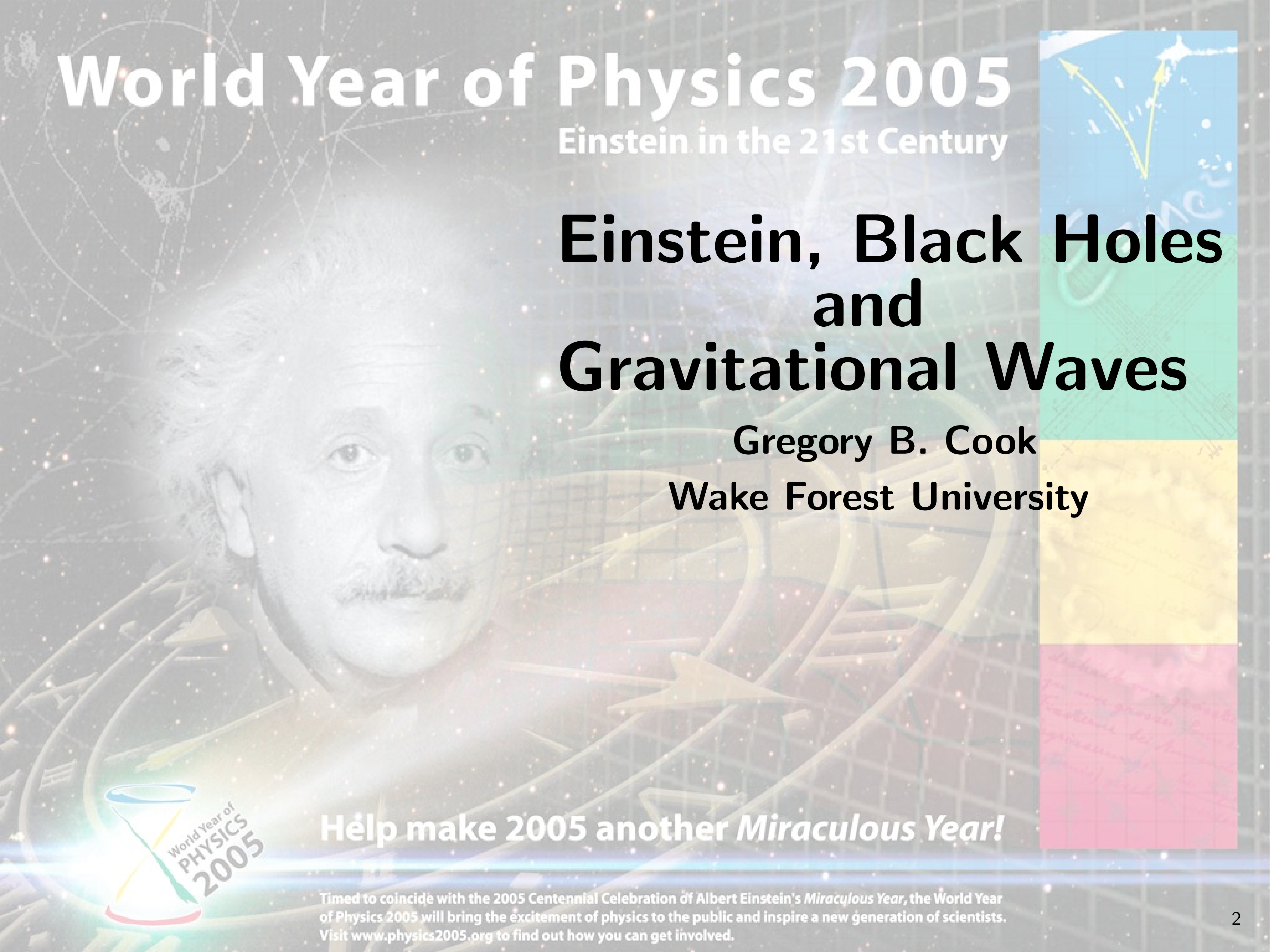
World Year of Physics 2005

Einstein in the 21st Century



Help make 2005 another *Miraculous Year!*

Timed to coincide with the 2005 Centennial Celebration of Albert Einstein's *Miraculous Year*, the World Year of Physics 2005 will bring the excitement of physics to the public and inspire a new generation of scientists. Visit www.physics2005.org to find out how you can get involved.



Einstein, Black Holes and Gravitational Waves

Gregory B. Cook

Wake Forest University

Einstein's Miraculous Year: 1905

- Einstein, A. "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt," *Annalen der Physik* **17** (1905) pp. 132–148.
  On a Heuristic Viewpoint Concerning the Production and Transformation of Light.

- Einstein, A. "Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen," *Annalen der Physik* **17** (1905) pp. 549–560.
  On the Motion—Required by the Molecular Kinetic Theory of Heat—of Small Particles Suspended in a Stationary Liquid.

* Einstein, A. "Zur Elektrodynamik bewegter Körper," *Annalen der Physik* **17** (1905) pp. 891–921.
  On the Electrodynamics of Moving Bodies.

- Einstein, A. "Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?" *Annalen der Physik* **18** (1905) pp. 639–641.
  Does the Inertia of a Body Depend Upon Its Energy Content?

- Einstein's doctoral dissertation at University of Zurich, published 1906.
  application of kinetic theory to the physical properties of solute sugar molecules

Goals For This Talk

- What is Einstein's Theory of General Relativity?
- What are some of the consequences of GR?
- What are Black Holes like and do they exist?
- What can we learn from Gravity Waves?
- To do all of this by analogy with Electrodynamics.

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Using Maxwell to Shed Some Light on Einstein

Gallilean Relativity

- Laws of physics look and work the same in any inertial frame.
- There is a universal time that all observers agree on.
- Velocities add "normally" as 3-vectors.

From Newton until the early 20th century, mainstream physics was built upon the foundation of Gallilean Relativity

Electrostatics and Newtonian Gravity

Coulomb electric field

$$
\begin{aligned}
\vec{F}_e &= k_e \frac{qQ}{r^2}\hat{r} \\
\vec{F}_e &= q\vec{E} \\
\vec{\nabla}\cdot\vec{E} &= 4\pi k_e \rho_c \\
\vec{E} &= -\vec{\nabla}\varphi \qquad \vec{\nabla}\times\vec{E} = 0 \\
\nabla^2\varphi &= -4\pi k_e \rho_c
\end{aligned}
$$

Newtonian gravity field

$$
\begin{aligned}
\vec{F}_m &= -G\frac{mM}{r^2}\hat{r} \\
\vec{F}_m &= m\vec{g} \\
\vec{\nabla}\cdot\vec{g} &= -4\pi G \rho_m \\
\vec{g} &= \vec{\nabla}\Psi \qquad \vec{\nabla}\times\vec{g} = 0 \\
\nabla^2\Psi &= -4\pi G \rho_m
\end{aligned}
$$

The "Problem" with Maxwell's Equations

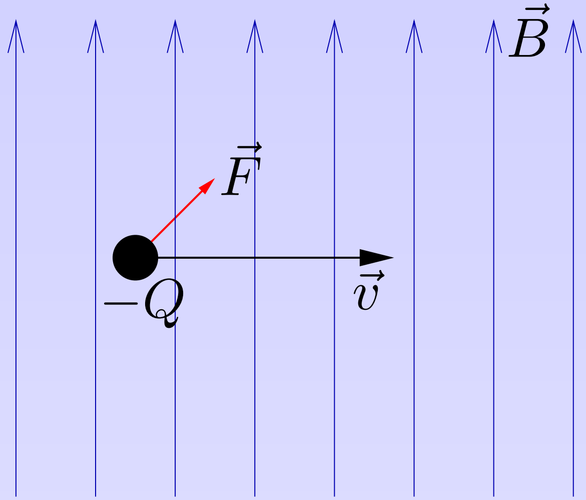
$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}\rho_c$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_c + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- No more "action at a distance"!
- In what frame does the EM wave have speed c?
- Maxwell's equations violate Gallilean Relativity!



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

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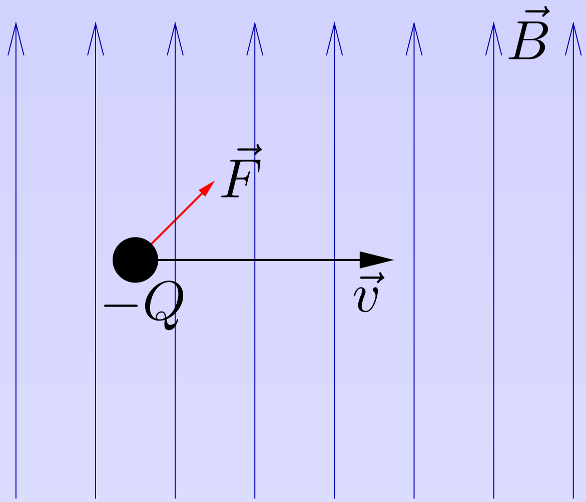

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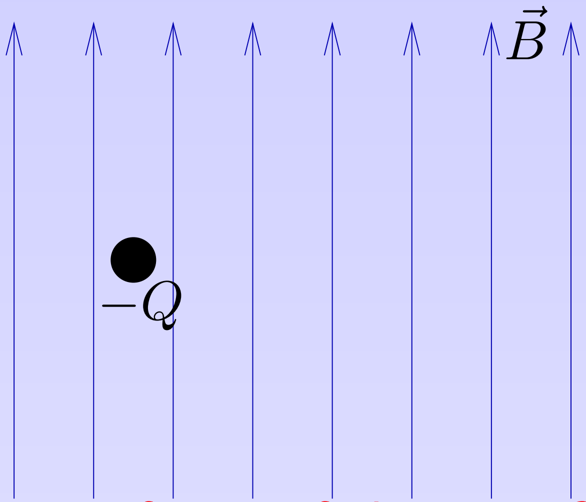

rest frame of charge $-Q$
No Force!

Special Relativity

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- The speed of light is a universal constant for *all* inertial observers.

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Consequences of Special Relativity:

- There is no preferred time – “relativity of simultaneity”
- 4-dimensional spacetime
- Time dilation
- Length contraction
- c is a limiting speed
- $E = mc^2$

E&M is Fixed – but Gravity is Broken!

Moving masses cause changes in the Newtonian gravitational field everywhere – *instantly*. Special Relativity forbids this!

- Coulomb's law had the same "problem", but the full theory of E&M (Maxwell's equations) fixes it — changes propagate as waves.

- We need a "richer" theory of gravity that allows changes in the gravitational field to propagate at finite speed. But how do we find it?

Einstein's solution was remarkably simple —

The Principle of Equivalence

- The effects of a gravitational field and a constant acceleration are indistinguishable.

General Relativity

Special Relativity + the Principle of Equivalence lead us directly to view gravity as the "natural motion" of matter and energy through a curved 4-D spacetime.

(Newtonian potential) $\Psi \quad \Longrightarrow \quad g_{\mu\nu}$ (spacetime metric)

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Maxwell's Equations — Einstein's Equations

$\nabla_\nu F^{\mu\nu} = \mu_0 J^\mu$ and $G_{\mu\nu} = 8\pi T_{\mu\nu}$

Maxwell's & Einstein's Equations

Maxwell's Equations
potential formulation

$$\vec{B} \equiv \vec{\nabla} \times \vec{A}$$

$$\nabla^2 \varphi = -\frac{\partial}{\partial t}\left(\vec{\nabla} \cdot \vec{A}\right) - \frac{1}{\epsilon_0}\rho_c$$

$$\frac{\partial}{\partial t}\vec{A} = -\vec{\nabla}\varphi - \vec{E}$$

$$\frac{\partial}{\partial t}\vec{E} = -\frac{1}{\mu_0\epsilon_0}\nabla^2\vec{A} - \frac{1}{\epsilon_0}\vec{J}_c + \frac{1}{\mu_0\epsilon_0}\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)$$

Coulomb gauge: $\vec{\nabla} \cdot \vec{A} = 0$

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Einstein's Equations
maximal slicing

$$\nabla^2\psi = \tfrac{1}{8}\psi R - \tfrac{1}{8}\psi^{-7}A_{ij}A^{ij} - 2\pi\psi^5\rho_m$$

$$\nabla_j A^{ij} = 8\pi\psi^{10}J^i_m$$

$$\frac{\partial}{\partial t}g_{ij} = -2\alpha\psi^{-6}A_{ij}$$

$$\begin{aligned}\frac{\partial}{\partial t}A_{ij} = {} & \alpha\psi^2\left(R_{ij} - \tfrac{1}{3}g_{ij}R\right) - 2\alpha\psi^{-6}A^m_i A_{jm} \\ & - \nabla_i\nabla_j(\alpha\psi^2) + \tfrac{1}{3}g_{ij}\nabla^2(\alpha\psi^2) \\ & + 8\alpha\psi^2(\nabla_i \ln\psi)\nabla_j \ln\psi \\ & - \tfrac{8}{3}g_{ij}\alpha\psi^2\left(\nabla^k \ln\psi\right)\nabla_k \ln\psi \\ & - 8\pi\alpha\psi^2\left(S_{ij} - \tfrac{1}{3}\psi^4 g_{ij}S\right)\end{aligned}$$

$$\nabla^2(\alpha\psi) - \tfrac{1}{8}\alpha\psi R - \tfrac{7}{8}\alpha\psi^{-7}A_{ij}A^{ij} = 0$$

Relativistic Gravity

How does General Relativity change our view of gravity?

- Moving masses cause changes in "gravity" that propagate with speed c.
  - Gravitational waves travel through spacetime.
- All forms of matter *and energy* are affected by gravity.
  - Light rays are deflected by massive bodies.
  - Light *redshifts* as it escapes a gravitating body.
- Orbits don't "close" because we no longer have a $\frac{1}{r}$ potential.
- Time slows down in a strong gravitational field.
- Rotating masses drag space and time with them.
- Regions of space become disconnected — Black Holes.

What is a Black Hole?

Short answer: When enough matter and energy are compressed into a small enough volume, gravity becomes so strong that nothing, not even light, can escape.

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A similar idea, discussed by Laplace, applies in Newtonian gravity when you let the escape velocity be the speed of light:

$$G\frac{mM}{R} = \frac{1}{2}mv_e^2 \quad : \quad v_e = c \quad \Rightarrow \quad R = \frac{2GM}{c^2}.$$

This is the radius of the event horizon for a Schwarzschild black hole! Right result, but *wrong idea*. This is not how a black hole behaves.

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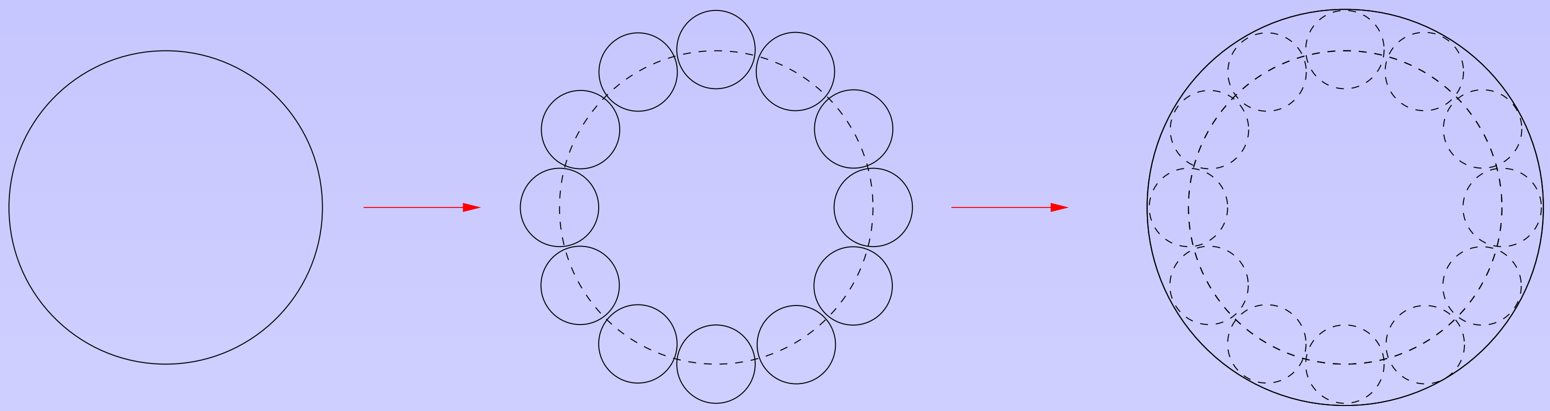
How does a black hole behave?

Trapped Regions

Consider a spherical electromagnetic wavefront – Huygens' Principle

— Area of outgoing wavefront expands.



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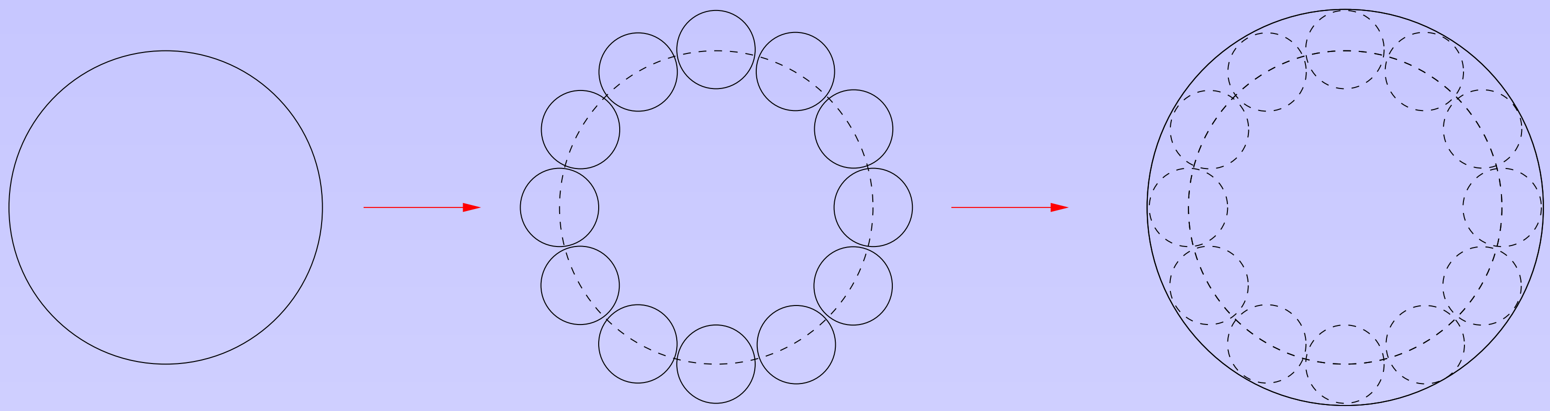
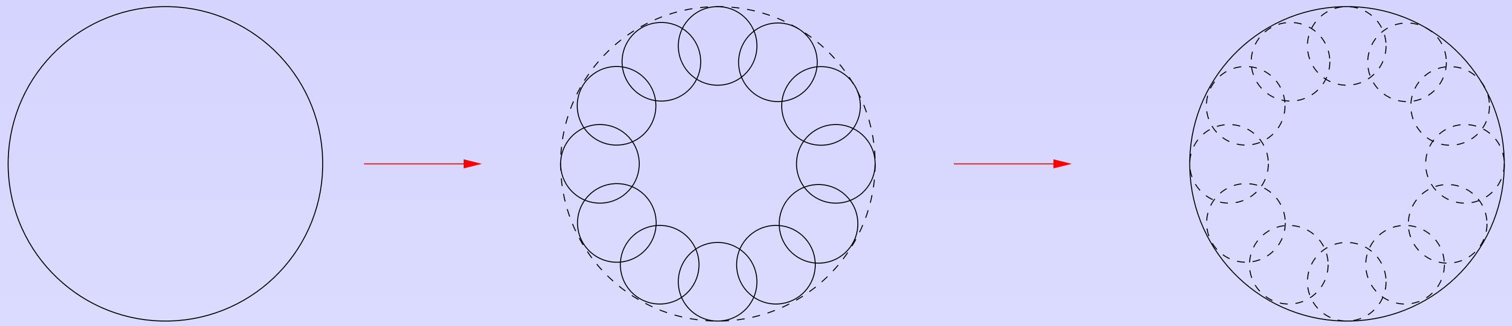
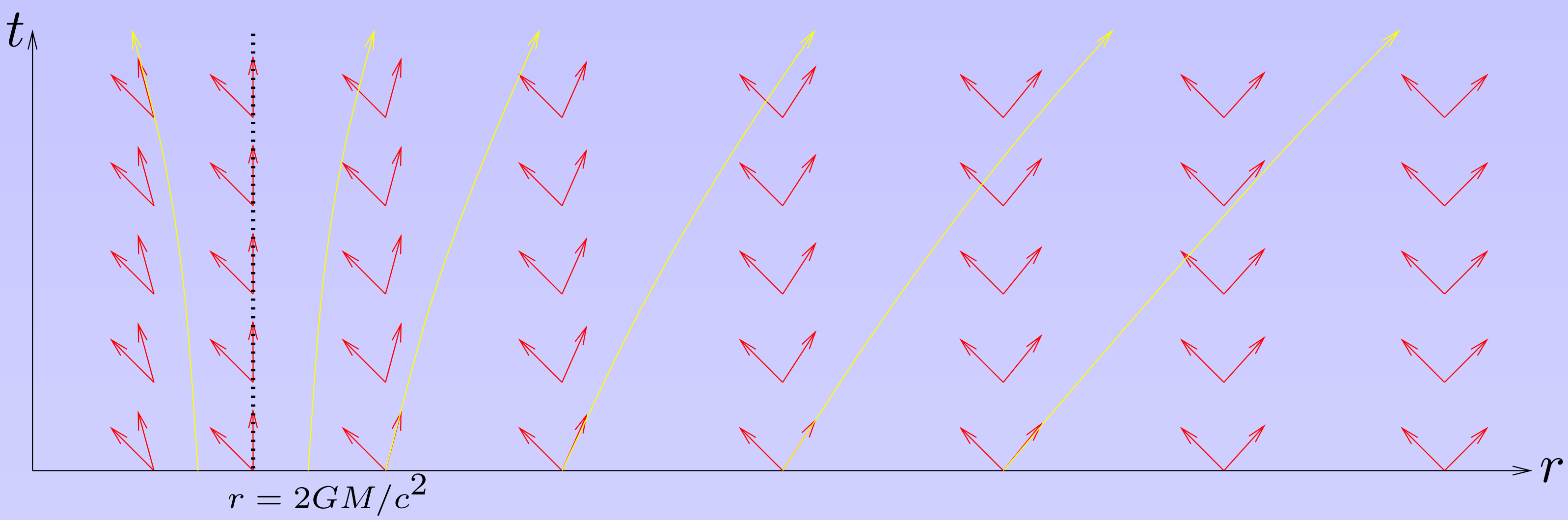


Gravity retards the expansion of outgoing wavefronts.
— A region is trapped if the *outgoing* wavefronts have no expansion or contract.



Black Holes & Light Cones

The effect of gravity on the expansion of wavefronts is to cause light cones to "tip".



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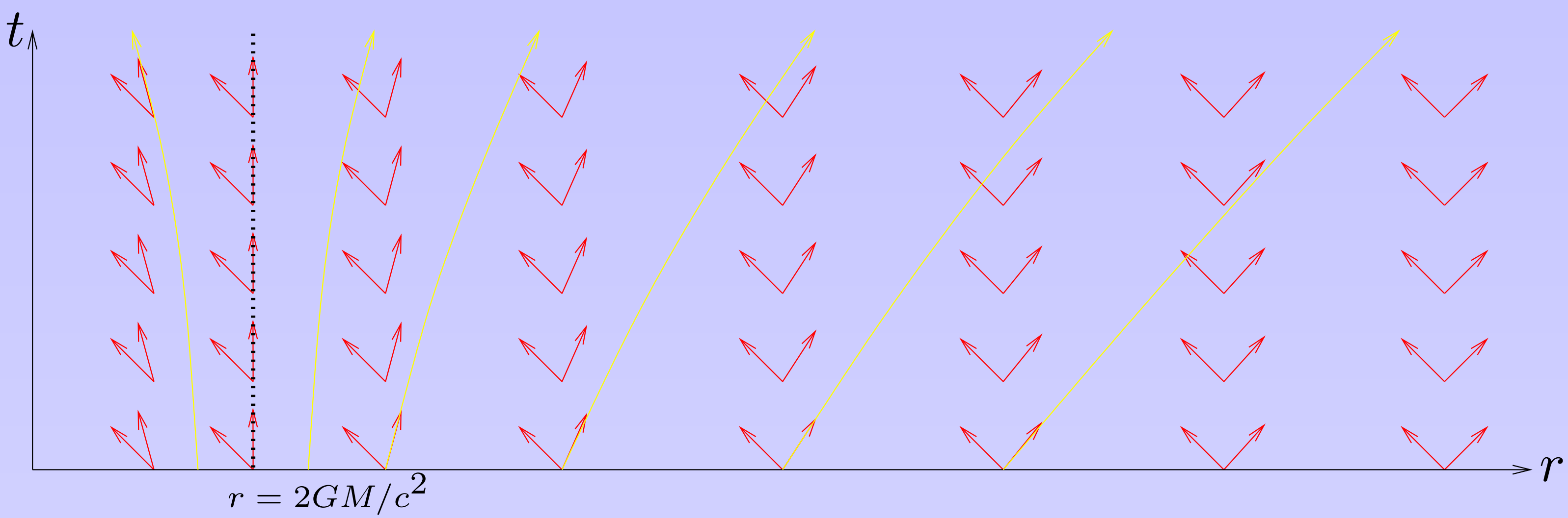

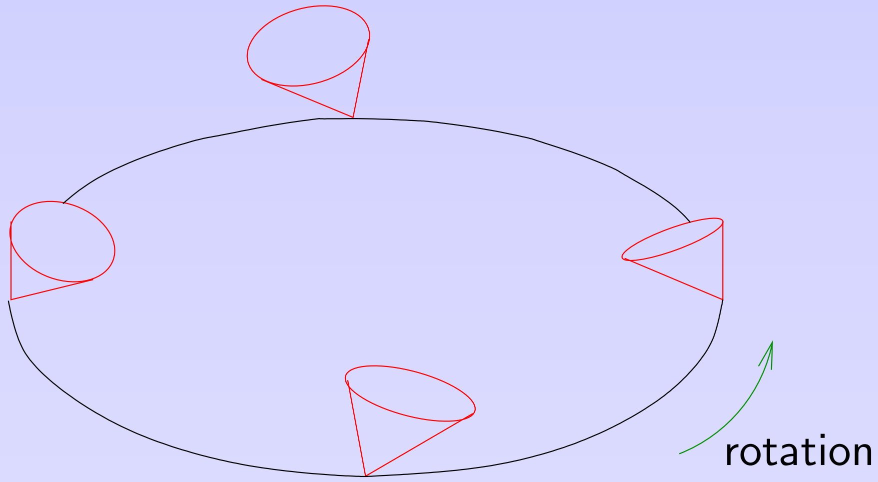

frame dragging

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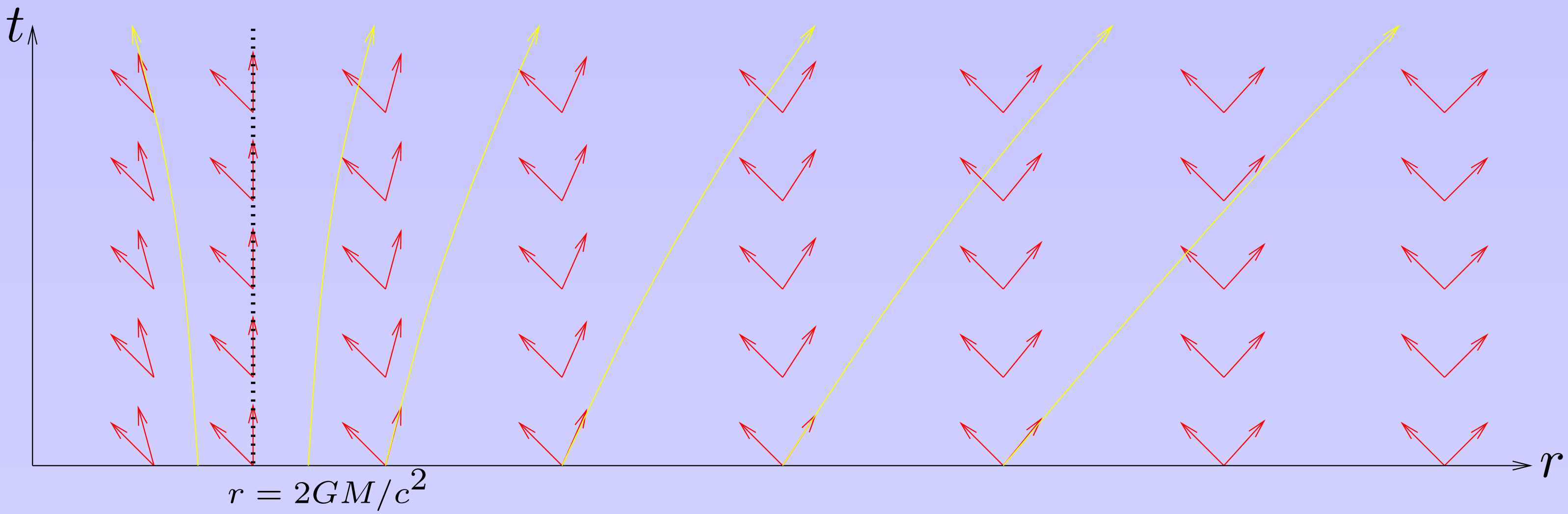

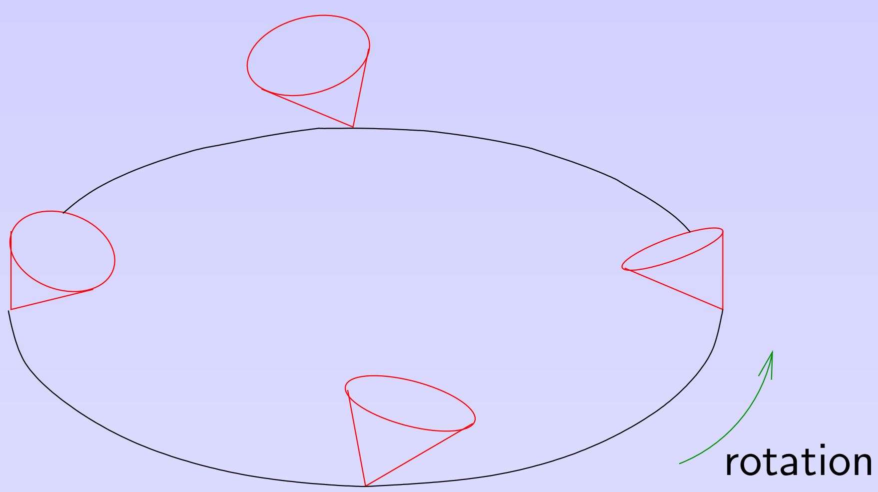

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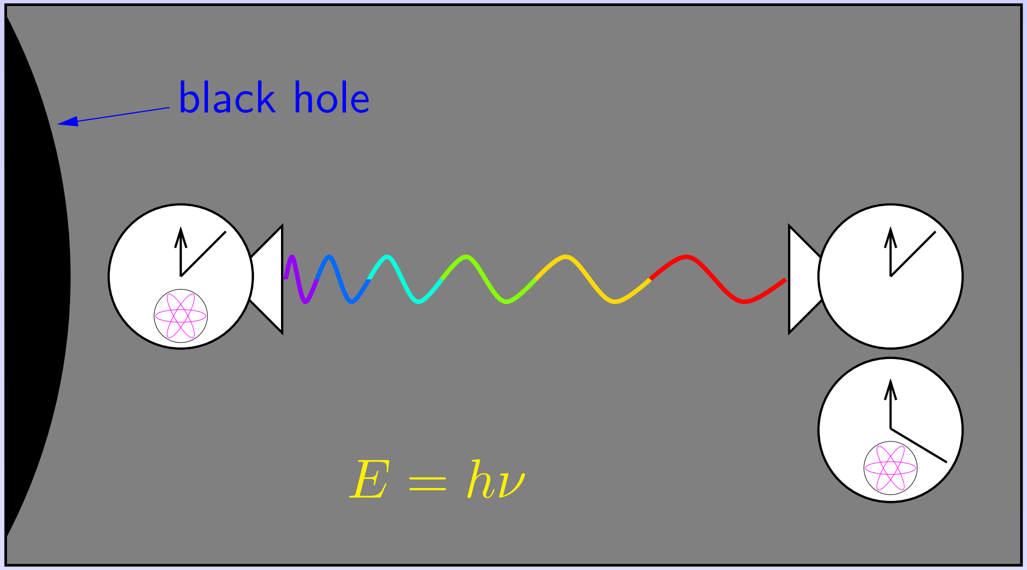
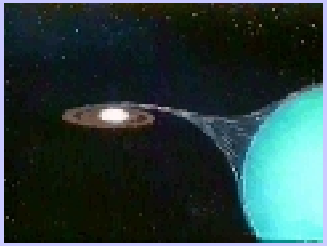


Do Black Holes Exist?

We think so, but we don't know for sure. We believe we have found them in many places, but by their very nature they are difficult to detect directly.

- We believe several exist in "X-ray Binaries" as the invisible companion of an ordinary star.
  - (Orbital period $\Rightarrow$ total mass) $- M_{\text{visible}}$
    $\Longrightarrow$ lower limit on $M_{\text{invisible}}$



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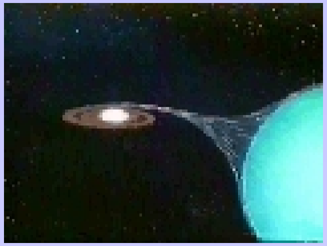
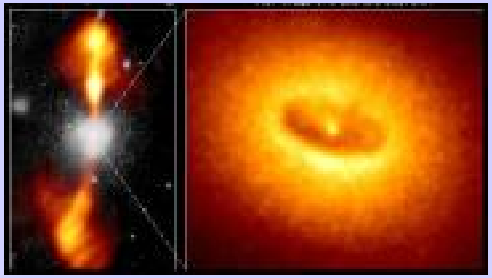


- We believe massive black holes exist at the centers of many, if not all, galaxies (AGN,quasar).
  - Energetics & Doppler measurements



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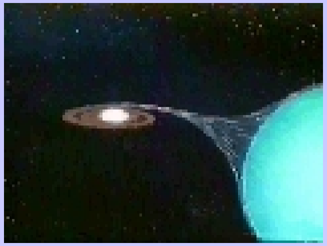

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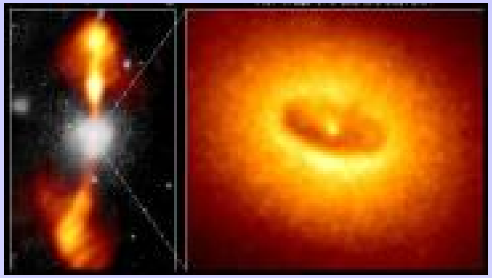
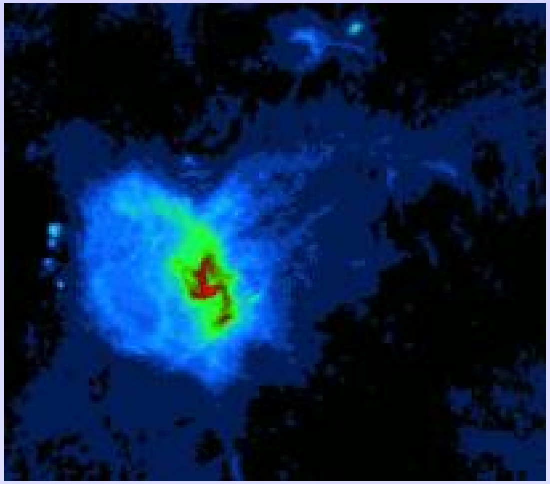


- We believe a $2.61 \times 10^6 M_\odot$ black hole lives at the center of our galaxy – Sag.A*.
  - mm-VLBI measurements have localized Sag.A* to 2AU ($\sim 20 R_{\rm BH}$).



Do Black Holes Exist?

- Every piece of evidence regarding the existence of black holes comes from EM radiation from matter in the vicinity of a black hole.
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  - Determine that the object is too dark or occupies too small a volume of space to be anything but a black hole.

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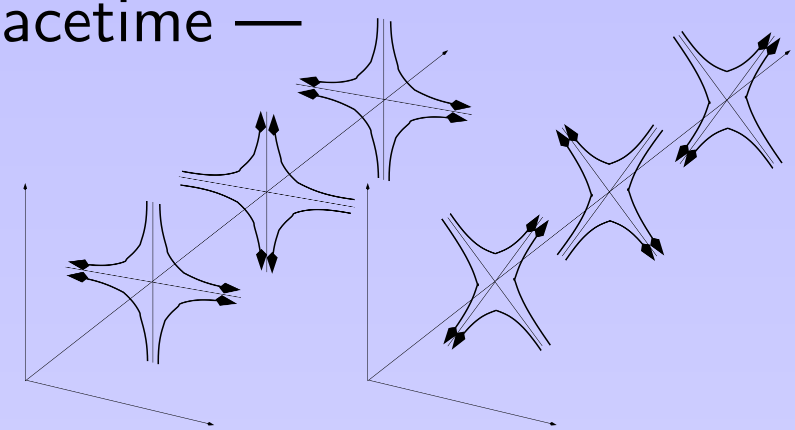
- While difficult, all cases can be refuted based on physics we may not yet fully understand.

- The convincing evidence we lack is a direct gravitational signature of a black hole.
  - Quasinormal mode ringing of a Black Hole, seen in *Gravitational Waves*.

What are Gravitational Waves

Propagating ripples in the fabric of spacetime —



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- Like E&M, waves are *transverse*
- Like E&M, there are two polarizations

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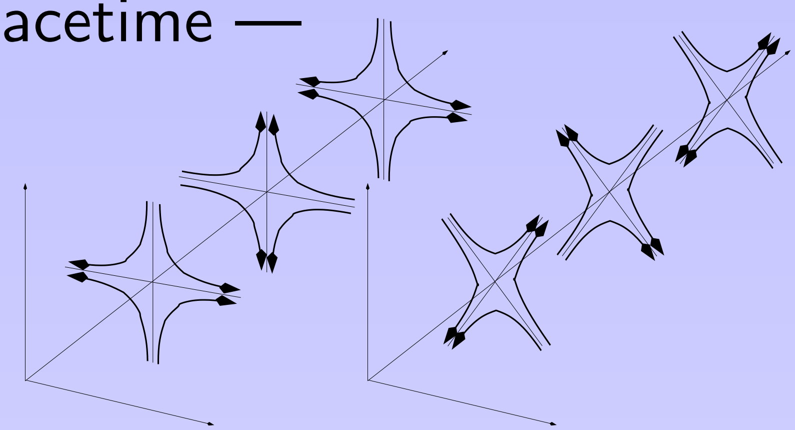

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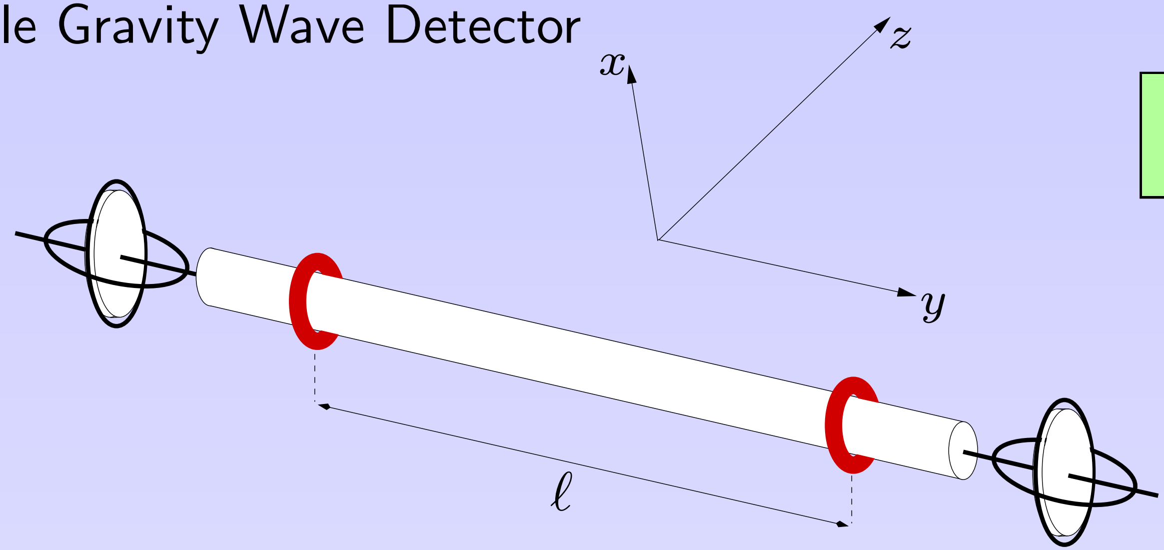
Simple Gravity Wave Detector



$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(t-z)$$

$$\ell = \ell_0 \Big\{ 1 + \tfrac{1}{4}\left[\bar{h}_{xx}(t-z) - \bar{h}_{yy}(t-z)\right] \sin^2\theta \cos 2\varphi + \tfrac{1}{2}\bar{h}_{xy}(t-z) \sin^2\theta \sin 2\varphi \Big\}$$

Gravitational Wave Sources

- Compact binary inspiral—chirps
- Supernovae/GRBs—bursts
- Pulsars in our galaxy—periodic
- Cosmological Signals—stochastic background

$I_{\mu\nu} \equiv$ mass quadrupole moment

$$h_{\mu\nu} = \frac{2}{r}\frac{G}{c^4}\frac{d^2}{dt^2}(I_{\mu\nu})$$

$1.4 M_\odot$ NS-NS binary, 20km sep.

$$h \approx \frac{10^{-21}}{(r/15\mathrm{Mpc})}$$

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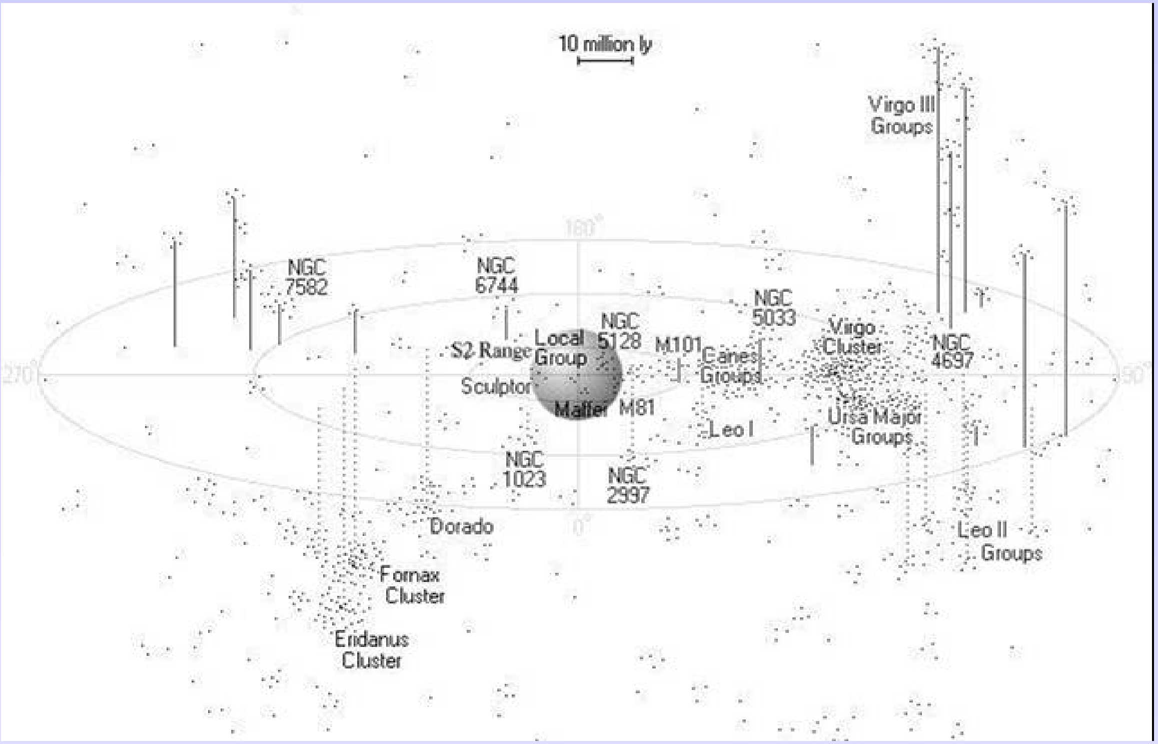
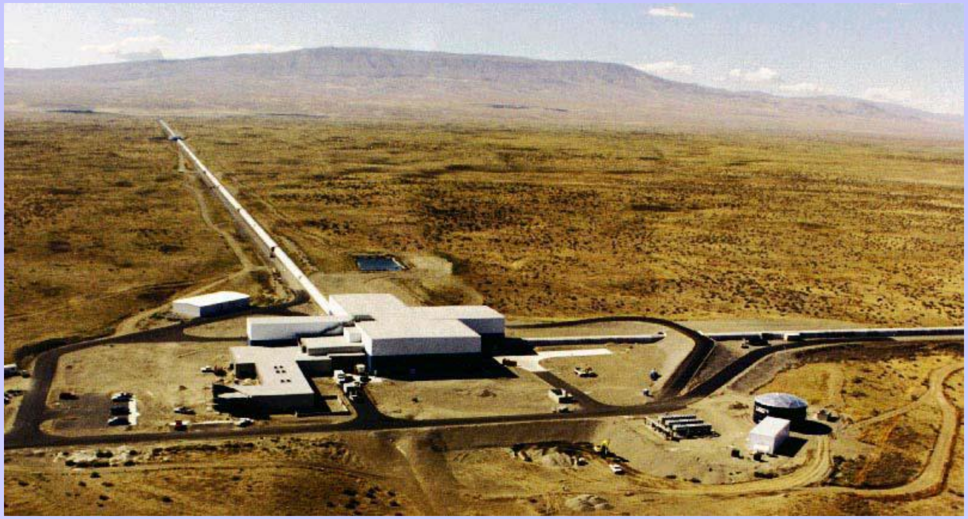


To see such events at a rate of a few per year to a few per day, searching a volume of space out to a radius of over 1Gpc, requires the ability to detect $h \sim 10^{-21}$!

Laser Interferometer Gravitational Wave Observatory

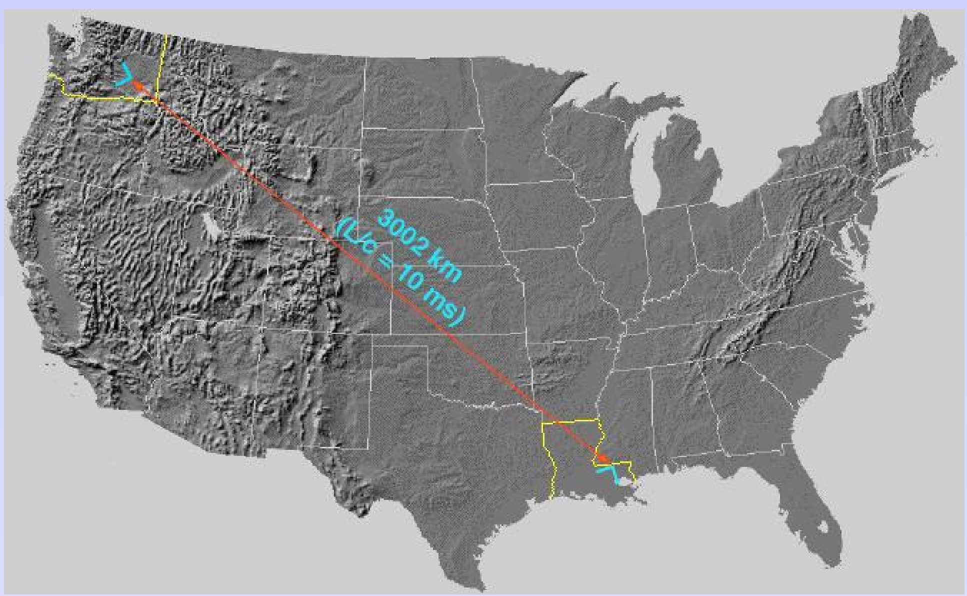
How do we measure $h = \frac{\Delta\ell}{\ell} \sim 10^{-21}$?



atomic diameter $\sim 10^{-10}$m

nuclear diameter $\sim 10^{-15}$m

$10^{-21} \times 4$km $\sim 10^{-18}$m



Source Simulation

Understanding the signals from LIGO

— making the connection to the astrophysical source —

means we need analytic and numerical models of all of the sources.

Compact Binary Coalescence

- Inspiral via secular radiative loss of energy and angular momentum.
  - Well understood via analytic PN calculations
- Dynamical plunge and coalescence.
  - Poorly known. Requires numerical simulations.
- Quasinormal mode ringdown.
  - Known from perturbative calculations

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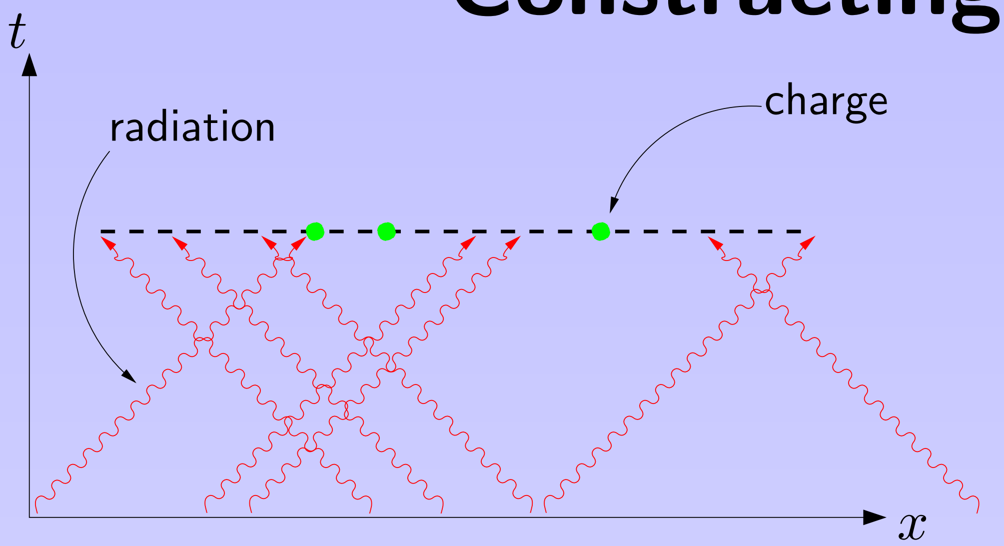
BH-BH Initial Data

What is required to describe gravitational initial conditions in General Relativity?

Constructing Initial Data



Electrodynamics

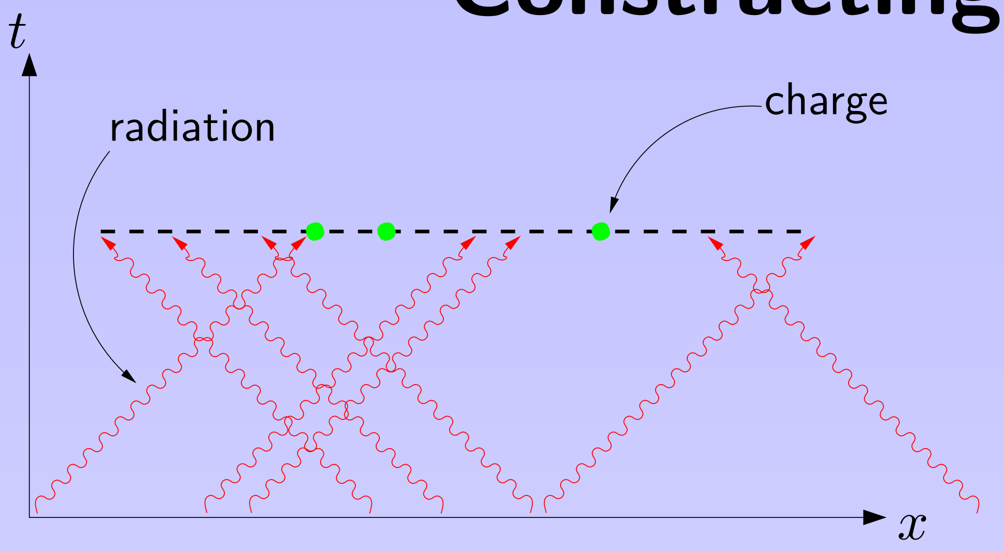
$$\vec{E} \equiv \vec{E}_{\mathsf{T}} - \vec{\nabla}\varphi \quad \text{with} \quad \vec{\nabla}\cdot\vec{E}_{\mathsf{T}} = 0$$

$$\nabla^2\varphi = -\frac{1}{\epsilon_0}\rho_c$$

$$\frac{\partial \vec{A}}{\partial t} = -\vec{E}_{\mathsf{T}} \qquad \text{Lorentz gauge}$$

$$\frac{\partial \vec{E}_{\mathsf{T}}}{\partial t} = -\frac{1}{\mu_0\epsilon_0}\nabla^2\vec{A} - \frac{1}{\epsilon_0}\vec{J}_c$$

Constructing Initial Data



Choose $\vec{A}$, $\vec{E}_{\mathsf{T}}$, and ρ_c.
Compute φ to satisfy Gauss's Law

Electrodynamics

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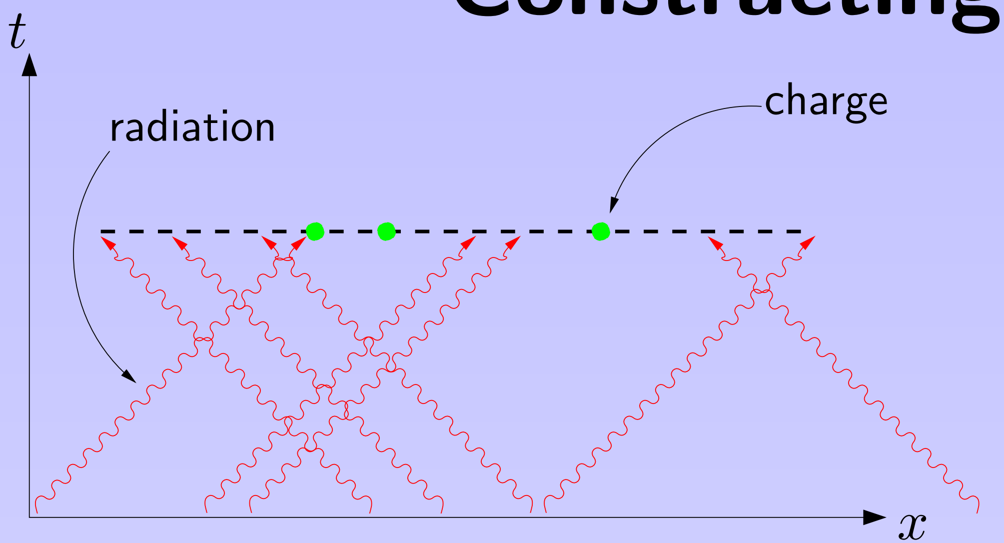
Constructing Initial Data



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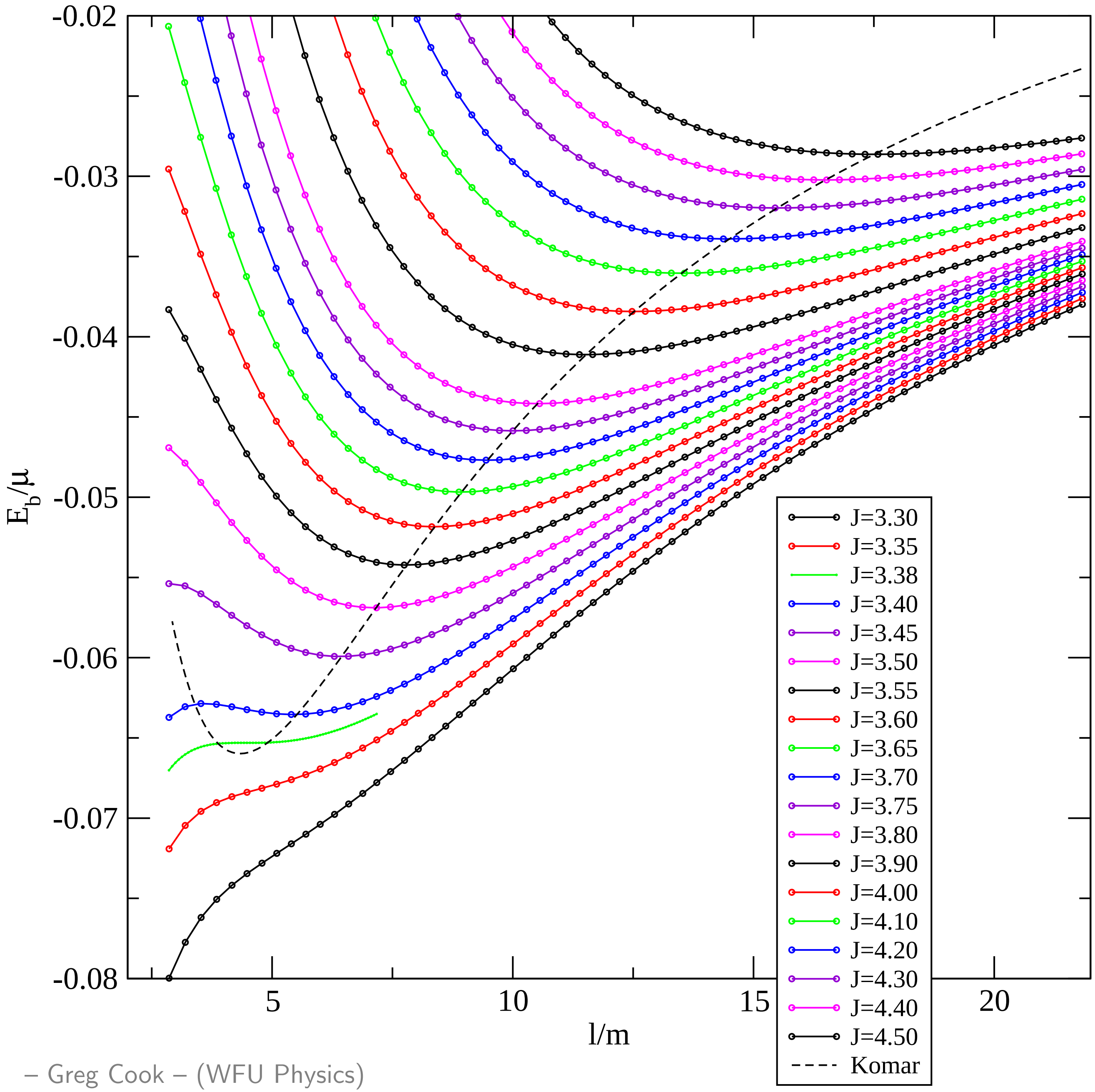
— Initial data in GR is treated the same way —

Break g_{ij} and A_{ij} into pieces that represent the incoming radiation, gauge freedom, and *constrained data*.

In GR, to get the constrained data, we must solve at least 4 equations similar to Gauss's law, but the equations are nonlinear and coupled.

Equal-Mass BH-BH Binary



Newtonian Effective Potential

$$\frac{V_{\text{eff}}}{\mu c^2} = \frac{1}{2c^2}\left(\frac{m}{r}\frac{J}{\mu m}\right)^2 - \frac{G}{c^2}\frac{m}{r}$$

GR equivalent:

$$\frac{E_b}{\mu} \quad \text{at const.} \quad \frac{J}{\mu m}$$

Minima –circular orbits– do not exist for all J!

Data: M. Caudill & J. Grigsby

The End