

Classical Model: Practice Problems
Intermediate Macroeconomics
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Question 1 Consider the following static classical economy:

The Representative Household has preferences over consumption goods, Y^D , and labor, L^S , given by the utility function $U(Y^D, L^S) = \ln(Y^D) + \ln(1 - L^S)$.

The Representative Firm owns a technology represented by the production function, $Y^S = 2KL^D$. The level of capital, K , is fixed at $K = 2$, i.e. the firm only chooses how much labor, L^D , it will employ.

- a) Using the representative household's utility function, derive the marginal utility of consumption.
- b) Using the representative household's utility function, derive the marginal utility of labor.
- c) Using the representative firm's production function, derive the marginal product of labor.
- d) Define a competitive equilibrium for this economy. Be sure your definition is written in a precise manner.
- e) Calculate the equilibrium you defined in part d), i.e. solve for the values of $\frac{w^*}{p^*}$, L^{S*} , L^{D*} , Y^{D*} , and Y^{S*} .
- f) What is the equilibrium profit of the firm?

Now, introduce money into this model. Assume the money supply for the economy is $M^S = 2$ and the propensity to hold money is $k = 1$.

- g) Graph the aggregate supply curve for this economy. Clearly label all relevant objects on your graph.
- h) What is the formula for the aggregate demand curve for this economy?
- i) Using the aggregate supply and demand curves for the economy, i.e. parts g) and h), determine the equilibrium price level.

Question 2 Consider the following static classical economy with taxes:

The Representative Household has preferences over consumption goods, Y^D , and labor, L^S , given by the utility function $U(Y^D, L^S) = Y^D - \frac{1}{2}(L^S)^2$. Suppose the representative household's labor income, wL^S , is taxed by the government at a tax rate, τ , where $0 \leq \tau \leq 1$. If $\tau = 0$, the government does not collect taxes. If $\tau = 1$, the government collects all the representative household's labor income as taxes. Assume government waste. After taxes are collected, the household's labor income is $wL^S(1 - \tau)$.

The Representative Firm owns a technology represented by the production function, $Y^S = 5L^D - \frac{1}{2}(L^D)^2$.

- a) Using the representative household's utility function, derive the marginal utility of consumption.
- b) Using the representative household's utility function, derive the marginal utility of labor.
- c) Using the representative firm's production function, derive the marginal product of labor.
- d) What is the formula for the representative household's budget constraint?
- e) Define the representative household's problem. Using this problem, calculate the labor supply curve for this economy, i.e. solve for L^S .
- f) Define the representative firm's problem. Using this problem, calculate the labor demand curve for this economy, i.e. solve for L^D .
- g) Using parts e) and f), solve for the equilibrium real wage, $\frac{w^*}{p^*}$, as a function of the tax rate, τ .
- h) Using part g) and either part e) or part f), solve for the equilibrium quantity of labor, L^* , as a function of the tax rate, τ . Note: $L^{S*} = L^{D*} = L^*$.
- i) Write an expression for the real tax revenue in this economy in terms of τ , $\frac{w^*}{p^*}$, and L^* .
- j) Derive the Laffer Curve for this economy, i.e. rewrite your expression for the real tax revenue from part i) in terms of only the tax rate, τ .
- k) Explain the position taken by "supply-side economist" in the context of the Laffer Curve.

Question 3 Consider the following static classical economy with taxes and transfers:

The Representative Household has preferences over consumption goods, Y^D , and labor, L^S , given by the utility function $U(Y^D, L^S) = Y^D - \frac{1}{2}(L^S)^2$. Suppose the representative household receives lump sum transfers, T , from the government. These transfers could include things like social security payments.

The Representative Firm owns a technology represented by the production function, $Y^S = 3L^D$. The government taxes the representative firm's revenue at the tax rate, τ , where $0 \leq \tau \leq 1$.

- a) Assuming the government balances its budget, write down an equation representing the government budget constraint.
- b) Define the representative household's problem. Using this problem, calculate the labor supply curve for this economy.
- c) Define the representative firm's problem. Using this problem, calculate the labor demand curve for this economy.
- d) Using parts a), b), and c), calculate the equilibrium for this economy, i.e. solve for the values of $\frac{w^*}{p^*}$, L^{S*} , L^{D*} , Y^{D*} , and Y^{S*} in terms of τ .
- e) What happens to household consumption in equilibrium, Y^{D*} , if the government decides to completely eliminate its tax on firm revenue? Show as much of your answer graphically as possible, and explain each step of your logic.

Question 4 Consider the following static classical economy:

The Representative Household has preferences over consumption goods, Y^D , and labor, L^S , given by the utility function $U(Y^D, L^S) = 2 \ln Y^D + 4 \ln(1 - L^S)$.

The Representative Firm owns a technology represented by the production function, $Y^S = 2KL^D$. The level of capital, K , is fixed at $K = 2$, i.e. the firm only chooses how much labor, L^D , it will employ.

a) Define the representative household's problem. Using this problem, calculate the labor supply curve for this economy.

b) Define the representative firm's problem. Using this problem, calculate the labor demand curve for this economy.

c) Using parts a) and b), calculate the equilibrium for this economy, i.e. solve for the values of $\frac{w^*}{p^*}$, L^{S*} , L^{D*} , Y^{D*} , and Y^{S*} .