Classical Model: Practice Problems Key
Intermediate Macroeconomics
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Question 1

a) The marginal utility of consumption measures how utility changes with changes in the quantity of consumption. As a result, the first derivative of the utility function w.r.t. consumption tells us the marginal utility of consumption: $\frac{\partial U(Y^D, L^S)}{\partial Y^D} = \frac{1}{Y^D}$.

b) The marginal utility of labor measures how utility changes with changes in the quantity of labor supplied. As a result, the first derivative of the utility function w.r.t. labor supplied can be used to derive the marginal utility of labor: $\frac{\partial U(Y^D, L^S)}{\partial L^S} = -\frac{1}{1-L^S}$.

c) The marginal product of labor measures how output changes with changes in the quantity of labor demanded. As a result, the first derivative of the production function w.r.t. labor demanded tells us the marginal product of labor: $\frac{\partial Y^S}{\partial L^D} = 4$.

d) A competitive equilibrium in the classical model is prices (p^*, w^*) and allocations $(L^{D*}, L^{S*}, Y^{D*}, Y^{S*})$ such that the following conditions hold:

1) Given prices (p^*, w^*), the representative household solves

$$\max_{Y^D, L^S} U(Y^D, L^S) \quad s.t. \quad p^* Y^D = w^* L^S$$

2) Given prices (p^*, w^*), the representative firm solves

$$\max_{Y^S, L^D} p^* Y^S - w^* L^D \quad s.t. \quad Y^S = f(L^D)$$

3) Markets clear

$$Y^{S*} = Y^{D*}$$
$$L^{S*} = L^{D*}$$

e) Labor demand is derived from the firm's problem.

$$\max_{Y^S, L^D} pY^S - wL^D \quad s.t. \quad Y^S = 4L^D$$

$$\Rightarrow \max_{L^D} 4pL^D - wL^D$$

$$\mathcal{L} = 4pL^D - wL^D$$

$$\frac{\partial \mathcal{L}}{\partial L^D} = 4p - w = 0$$

$$\frac{w}{p} = 4$$

This is an elastic labor demand curve, so $\frac{w^*}{p^*} = 4$.
Labor supply is derived from the household's problem.

$$\max_{Y^D, L^S} \ln(Y^D) + \ln(1 - L^S) \quad s.t. \quad pY^D = wL^S$$

$$\Rightarrow \max_{L^S} \ln\left(\frac{w}{p}L^S\right) + \ln(1 - L^S)$$

$$\mathcal{L} = \ln\left(\frac{w}{p}L^S\right) + \ln(1 - L^S)$$

$$\frac{\partial \mathcal{L}}{\partial L^S} = \frac{1}{\frac{w}{p}L^S} \cdot \frac{w}{p} - \frac{1}{1 - L^S} = 0$$

Solving for L^S,

$$L^S = \frac{1}{2}$$

This is an inelastic labor supply curve, so $L^{S*} = \frac{1}{2}$. Using the market clearing condition for the labor market, $L^{D*} = L^{S*} = \frac{1}{2}$. Using the firm's production function, $Y^{S*} = 4(L^{D*}) = 4(\frac{1}{2}) = 2$. Finally, using the market clearing condition for the goods market, $Y^{D*} = Y^{S*} = 2$.

The competitive equilibrium for this example is the price $\frac{w^*}{p^*} = 4$ and allocations $L^{S*} = \frac{1}{2}, L^{D*} = \frac{1}{2}, Y^{S*} = 2$, and $Y^{D*} = 2$.

f) Given the perfectly competitive environment, we should expect equilibrium profits to be zero, which proves to be true: $\pi^* = p^*Y^{S*} - w^*L^{D*} = (1)(2) - (4)(\frac{1}{2}) = 0$.

g) See Figure 1 below.

h) $p = \frac{M^S}{kY^D} = \frac{2}{Y^D}$

i) Using the aggregate supply curve, $Y^{S*} = Y^{D*} = 2$, which means $p^* = \frac{2}{2} = 1$.

Question 2

a) The marginal utility of consumption measures how utility changes with changes in the quantity of consumption. As a result, the first derivative of the utility function w.r.t. consumption tells us the marginal utility of consumption: $\frac{\partial U(Y^D, L^S)}{\partial Y^D} = 1$.

b) The marginal utility of labor measures how utility changes with changes in the quantity of labor supplied. As a result, the first derivative of the utility function w.r.t. labor supplied can be used to derive the marginal utility of labor: $\frac{\partial U(Y^D, L^S)}{\partial L^S} = -L^S$.

c) The marginal product of labor measures how output changes with changes in the quantity of labor demanded. As a result, the first derivative of the production function w.r.t. labor demanded tells us the marginal product of labor: $\frac{\partial Y^S}{\partial L^D} = 5 - L^D$.

d) If the government taxes household labor income at the rate τ, the household's take home pay is $wL^S(1-\tau)$, which means the household's budget constraint is $pY^D = wL^S(1-\tau)$.

e)

$$\max_{Y^D, L^S} Y^D - \frac{1}{2}(L^S)^2 \quad s.t. \quad pY^D = wL^S(1-\tau)$$

$$\Rightarrow \max_{L^S} \frac{w}{p}L^S(1-\tau) - \frac{1}{2}(L^S)^2$$

$$\mathcal{L} = \frac{w}{p}L^S(1-\tau) - \frac{1}{2}(L^S)^2$$

$$\frac{\partial \mathcal{L}}{\partial L^S} = \frac{w}{p}(1-\tau) - L^S = 0$$

$$L^S = \frac{w}{p}(1-\tau)$$

f)

$$\max_{Y^S, L^D} pY^S - wL^D \quad s.t. \quad Y^S = 5L^D - \frac{1}{2}(L^D)^2$$

$$\Rightarrow \max_{L^D} p\Big(5L^D - \frac{1}{2}(L^D)^2\Big) - wL^D$$

$$\mathcal{L} = p\Big(5L^D - \frac{1}{2}(L^D)^2\Big) - wL^D$$

$$\frac{\partial \mathcal{L}}{\partial L^D} = p5 - pL^D - w = 0$$

$$L^D = 5 - \frac{w}{p}$$

g) Using the market clearing condition in the labor market,

$$L^{S*} = L^{D*}$$

$$\frac{w^*}{p^*}(1-\tau) = 5 - \frac{w^*}{p^*}$$

$$\frac{w^*}{p^*} = \frac{5}{2-\tau}$$

h) Using the labor supply curve, $L^* = L^{S*} = \frac{w^*}{p^*}(1-\tau) = 5\frac{1-\tau}{2-\tau}$

OR

Using the labor demand curve, $L^* = L^{D*} = 5 - \frac{w^*}{p^*} = 5\frac{2-\tau}{2-\tau} - \frac{5}{2-\tau} = 5\frac{1-\tau}{2-\tau}$

i) Real Tax Revenue $= \frac{w^*}{p^*}L^*\tau$

j) Plugging in the equilibrium values for L^* and $\frac{w^*}{p^*}$ derived above, Real Tax Revenue $= \frac{5}{2-\tau}5\frac{1-\tau}{2-\tau}\tau = 25\frac{(1-\tau)}{(2-\tau)^2}\tau$.

k) "Supply-siders" argued the government should cut taxes to increase revenue. They believed U.S. tax rates were such that we were on the "wrong side" of the Laffer Curve, that is, to the right of the tax rate which generates the maximum amount of revenue.

Question 3

a) The balanced government budget shows total tax revenue equals total expenditure: $\tau p Y^S = T$.

b)

$$\max_{Y^D, L^S} Y^D - \frac{1}{2}(L^S)^2 \quad s.t. \quad pY^D = wL^S + T$$

$$\Rightarrow \max_{L^S} \frac{w}{p}L^S + \frac{T}{p} - \frac{1}{2}(L^S)^2$$

$$\mathcal{L} = \frac{w}{p}L^S + \frac{T}{p} - \frac{1}{2}(L^S)^2$$

$$\frac{\partial \mathcal{L}}{\partial L^S} = \frac{w}{p} - L^S = 0$$

$$L^S = \frac{w}{p}$$

c)

$$\max_{Y^S, L^D} (1-\tau)pY^S - wL^D \quad s.t. \quad Y^S = 3L^D$$

$$\Rightarrow \max_{L^D} (1-\tau)p3L^D - wL^D$$

$$\mathcal{L} = (1-\tau)p3L^D - wL^D$$

$$\frac{\partial \mathcal{L}}{\partial L^D} = (1-\tau)p3 - w = 0$$

$$\frac{w}{p} = 3(1-\tau)$$

d) Using the labor demand curve, $\frac{w^*}{p^*} = 3(1-\tau)$. Plugging the equilibrium real wage into the labor supply curve gives $L^{S*} = 3(1-\tau)$. Market clearing in the labor market implies $L^{D*} = L^{S*} = 3(1-\tau)$. Plugging the equilibrium quantity of labor demanded into the firm's production function gives $Y^{S*} = 9(1-\tau)$. Market clearing in the goods market then implies $Y^{D*} = Y^{S*} = 9(1-\tau)$. Notice, the goods market equilibrium can also be solved through the household's budget constraint:

$$pY^D = wL^S + T,$$

which we know also holds with equality in equilibrium, so we have the rewritten budget constraint

$$Y^{D*} = \frac{w^*}{p^*}L^{S*} + \frac{T}{p^*}.$$

Using the government budget constraint and plugging in the equilibrium values for the real wage and labor supplied gives the following:

$$Y^{D*} = 3(1-\tau)3(1-\tau) + \tau Y^{S*},$$

which simplifies to

$$Y^{D*} = 9(1-\tau).$$

By the market clearing condition, we know $Y^{S*} = Y^{D*} = 9(1-\tau)$.

e) See Figure 2 below. Household consumption in equilibrium, Y^{D*}, increases after the government eliminates the tax on total revenue. Eliminating the tax shifts the labor demand curve of the firm outward, which increases the equilibrium quantity of labor demanded, L^{D*}. Using the firm's production function, output supplied in equilibrium, Y^{S*}, also increases. Through the market clearing condition in the goods market, Y^{D*} must also increase.

Question 4

a)

$$\max_{Y^D,\, L^S} 2\ln Y^D + 4\ln(1-L^S) \quad s.t. \quad pY^D = wL^S$$

$$\max_{L^S} 2\ln\left(\frac{w}{p}L^S\right) + 4\ln(1-L^S)$$

$$\mathcal{L} = 2\ln\left(\frac{w}{p}L^S\right) + 4\ln(1-L^S)$$

$$\frac{\partial \mathcal{L}}{\partial L^S} = \frac{2}{\frac{w}{p}L^S}\cdot\frac{w}{p} - \frac{4}{1-L^S} = 0$$

$$\frac{2}{L^S} - \frac{4}{1-L^S} = 0$$

$$2 - 2L^S = 4L^S$$

$$L^S = \frac{1}{3}$$

b)

$$\max_{Y^S,\, L^D} p\cdot Y^S - wL^D \quad s.t. \quad Y^S = 4L^D$$

$$\max_{L^D} p4L^D - wL^D$$

$$\mathcal{L} = p4L^D - wL^D$$

$$\frac{\partial \mathcal{L}}{\partial L^D} = p4 - w = 0$$

$$\frac{w}{p} = 4$$

c) The labor supply curve is perfectly inelastic, which means $L^{S*} = \frac{1}{3}$. Using the market clearing condition for the labor market, $L^{D*} = \frac{1}{3}$. The labor demand curve is perfectly elastic, which means $\frac{w*}{p*} = 4$. Using the firm's production function, $Y^{S*} = 4L^{D*} = \frac{4}{3}$. Using the market clearing condition in the goods market, $Y^{D*} = \frac{4}{3}$. Notice, the goods market equilibrium can also be solved through the household's budget constraint:

$$pY^D = wL^S,$$

which we know also holds with equality in equilibrium, so we have the rewritten budget constraint

$$Y^{D*} = \frac{w^*}{p^*}L^{S*}.$$

Plugging in the equilibrium values for the real wage and labor supplied gives the following:

$$Y^{D*} = \frac{4}{3}.$$

By the market clearing condition, we know $Y^{S*} = Y^{D*} = \frac{4}{3}$.

Figure 1:

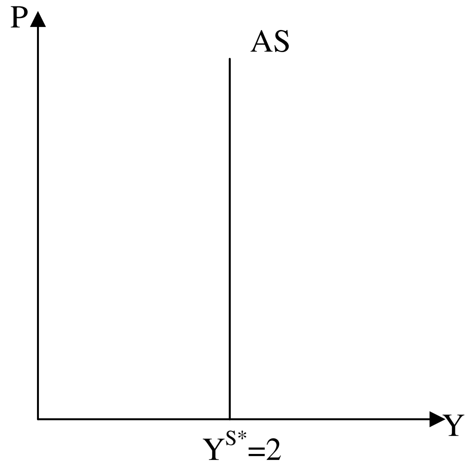

Figure 2: