

New Keynesian Unemployment: Practice Problem Key
Intermediate Macroeconomics
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Question 1

a) The firm chooses its efficiency wage by minimizing total costs.

$$\begin{aligned} \min_{\frac{w}{p}} \frac{w}{p} L^D + c \left(\frac{w}{p} \right) L^D \\ \Rightarrow \mathcal{L} = \frac{w}{p} L^D - \ln \left(\frac{w}{p} \right) L^D \\ \frac{\partial \mathcal{L}}{\partial \frac{w}{p}} = L^D - \frac{L^D}{\frac{w}{p}} = 0 \\ L^D = \frac{L^D}{\frac{w}{p}} \\ \Rightarrow \left(\frac{w}{p} \right)_{\text{eff}} = 1 \end{aligned}$$

Given the efficiency wage, the firm then maximizes its profit, which determines the labor demand function.

$$\begin{aligned} \max_{Y^S, L^D} Y^S - \left(\frac{w}{p} \right)_{\text{eff}} L^D - c \left(\left(\frac{w}{p} \right)_{\text{eff}} \right) L^D \quad s.t. \quad Y^S = 2(L^D)^{\frac{1}{2}} \\ \Rightarrow \mathcal{L} = 2(L^D)^{\frac{1}{2}} - L^D - \underbrace{\ln(1)L^D}_{=0 \text{ because } \ln(1)=0} \\ \frac{\partial \mathcal{L}}{\partial L^D} = (L^D)^{-\frac{1}{2}} - 1 = 0 \end{aligned}$$

$$L^D = 1$$

b) Given the labor supply function and the efficiency wage from part a), $L^S = 2 \left(\frac{w}{p}\right)_{\text{eff}} = 2(1) = 2$. Therefore, this economy's natural rate of unemployment $= L^S - L^D = 2 - 1 = 1$.

Question 2

a) From our discussion of the firm's cost minimization problem in lecture, we know the marginal benefit of increasing the real wage is equal to the negative of the derivative of the per worker turnover cost function w.r.t. the real wage, i.e. marginal benefit $= -c'(\frac{w}{p})$. In this case, $c'(\frac{w}{p}) = -4(\frac{w}{p})^{-2}$, so the marginal benefit $= 4(\frac{w}{p})^{-2}$.

b) The firm chooses its efficiency wage by minimizing total costs.

$$\begin{aligned} \min_{\frac{w}{p}} \frac{w}{p} L^D + c\left(\frac{w}{p}\right) L^D \\ \Rightarrow \mathcal{L} = \frac{w}{p} L^D + 4\left(\frac{w}{p}\right)^{-1} L^D \\ \frac{\partial \mathcal{L}}{\partial \frac{w}{p}} = L^D - 4\left(\frac{w}{p}\right)^{-2} L^D = 0 \\ 4\left(\frac{w}{p}\right)^{-2} = 1 \\ \Rightarrow \left(\frac{w}{p}\right)_{\text{eff}} = 2 \end{aligned}$$

c) To find the market-clearing real wage, which is the same as $\left(\frac{w}{p}\right)^*$ from the classical model, set the labor demand curve and labor supply curve equal to one another and solve for the real wage:

$$2 \left(\frac{w}{p}\right)^{-1} = \frac{w}{p}$$

$$2 = \left(\frac{w}{p}\right)^2$$

$$\Rightarrow \left(\frac{w}{p}\right)^* = 1.41,$$

which, as we would expect, is lower than the efficiency wage.