December 10, 2025 Name ________________

Final Exam PHY 310/610

This test consists of three parts. In parts II and III, PHY 310 students can skip one question of those offered, while PHY 610 students must answer all questions.

Part I: Multiple Choice (mixed new and review questions) [40 points] (2 points each)
PHY 310/610: For each question, choose the best answer

1. The first Friedmann equation says that $\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2}$. What is the quantity a?
   A) The distance to a particular galaxy, such as Andromeda
   B) The wavelength of a photon from the cosmic microwave background
   C) The distance you have to go to completely circle the universe
   D) The typical size of gaps between galaxies
   E) The "scale factor", to which all other distances are proportional

2. As the universe expands, radiation scales approximately as what power of a?
   A) a^{-4} B) a^{-3} C) a^{-2} D) $a^0 = 1$ E) a^4

3. The spectrum of the cosmic microwave background is what type of spectrum?
   A) Black body B) White noise C) Monochromatic D) Top Hat E) None of these

4. The isotope that is a "bottleneck" that prevents further steps of primordial nucleosynthesis until it happens is
   A) ^{3}He B) ^{4}He C) Deuterium, ^{2}H D) Tritium ^{3}H E) None of these

5. Which of the following do we not have a clear indication of why it is the way it is?
   A) Why the universe is almost homogenous
   B) Why the universe is almost isotropic
   C) Why the primordial abundance of Helium is about 25% by mass
   D) Why there is apparently more matter than anti-matter in the universe
   E) Why the cosmic microwave background has the spectrum it has

6. If you were to summarize general relativity in one to two sentences, what would it be?
   A) Matter causes other matter to accelerate, including relativistic effects
   B) Matter makes spacetime accelerate, and spacetime makes matter accelerate
   C) All curvature is illusion, which can be eliminated by working in accelerating coordinates
   D) Matter tells space how to curve, and space tells matter how to move
   E) Gravity causes objects to stop following geodesics, due to gravitational forces

7. When a galaxy collides with another galaxy, the most damage is done if the galaxy it collides with is ________ and the collision is ________ .
   A) Massive, fast B) Massive, slow C) Light, fast D) Light, slow E) I don't know

8. The reason there are giant elliptical galaxies often near the center of galaxy clusters is, we think, because
   A) The most massive galaxies attracted all the other galaxies in the cluster
   B) Gravitational lensing from the cluster makes it look bigger than it is
   C) These galaxies expelled their dark matter, but compensated with more ordinary matter
   D) Many galaxies merged to make these most massive galaxies
   E) These galaxies have expelled all the matter that would make it look dimmer

9. What event caused the photons to end up at a higher temperature than the neutrinos?
   A) Nucleosynthesis
   B) Proton/neutron freezeout
   C) Quark confinement
   D) Electroweak unification
   E) Electron-positron annihilation

10. The name of our galaxy is
    A) Milky Way B) Virgo C) Laniakea D) Coma E) Fornax

11. In Grand Unified theories (GUTs), the three forces of the standard model are unified into one at some very high energy. What is an additional likely prediction of GUTs?
    A) Neutrinos are massless
    B) Protons, even in hydrogen, are unstable
    C) Black holes should evaporate very slowly
    D) Dark matter is probably GUT scale particles
    E) None of these are common predictions of GUTs

12. Which of the following is <u>not</u> a name given to the potential shape of the universe due to curvature?
    A) Flat B) Hyperbolic C) Closed D) Open E) All of these are possibilities

13. Which of the following happened before proton/neutron freezeout?
    A) Recombination
    B) Matter/radiation equality
    C) Nucleosynthesis
    D) Quark confinement
    E) First structure

14. The age of the universe is approximately
    A) 13.8 Myr B) 138 Myr C) 1.38 Gyr D) 12.3 Gyr E) 13.8 Gyr

15. If we go back to very high red-shift, say $z = 10^4$, what do we think the value of Ω was?
    A) 0.2607 B) 0.9999 C) 1.0000 D) 1.0001 E) Unknown

16. Strong interactions can violate conservation of
    A) Energy B) Baryon Number C) Electric Charge D) Strangeness E) None of these

17. Which two forces are already combined in standard model particle physics?
    A) Electromagnetism and gravity
    B) Electromagnetism and the weak force
    C) The strong force and the weak force
    D) The strong force and electromagnetism
    E) Gravity and the weak force

18. The primary evidence for the presence of large quantities of dark matter in spiral galaxies comes from
    A) Noticing how dark they are
    B) Obscuration of galaxies behind them
    C) Rotation curves that do not fall off with distance
    D) Photons coming from annihilation of dark matter
    E) Comparing the mas of individual stars to the amount of baryonic matter in them

19. Which force is *not* included in the standard model of particle physics
    A) Weak B) Strong C) Gravity D) Electromagnetism E) These are all included

20. At modest red-shift, say $z = 4$, which component dominated the universe?
    A) Matter B) Radiation C) Neutrinos D) Dark energy E) Curvature

Part II Short Answer [40 points/50 points] (10 points each)
PHY 310: Choose **four** of the following five questions and give a short answer (1-3 sentences)
PHY 610: Answer **all five** questions

21. Hubble's law says that the velocity of a galaxy (as measured by its red-shift) is directly proportional to its distance. Give at least three reasons why this might be unreliable for very nearby galaxies of very distant galaxies.

22. At right are the five biggest components of the universe, and to its right are listed our best guess of Ω_i that each contributes. Match the items to their Ω_i values. You can just draw lines if you want.

Items	Ω_i
Ordinary matter	0.6889
Neutrinos	0.2607
Electromagnetic radiation	0.0490
Dark matter	0.0014
Dark energy	0.0001

23. Was the universe always transparent? If so, explain why, or if not, explain which step in the universe's evolution caused a transition (and was it transparent → opaque or vice versa). Don't just give the name of this step, describe what happened at that time.

24. Inflation is a hypothesized stage in the universe when the universe grew at an exponential rate in a very short time. Describe which three things inflation is supposed to explain.

25. List at least four future events that we discussed might occur of a catastrophic nature (*i.e.*, it could wipe out humanity)

Units and Constants	Physical Constants	Age of Universe	Distance / Magnitudes
$\text{pc} = 3.086\times10^{16}$ m	$k_B = 8.617\times10^{-5}$ eV/K	Matter: $t = \dfrac{17.3\text{ Gyr}}{(z+1)^{3/2}}$	$d = 10^{1+\frac{m-M}{5}}$ pc
$\text{eV} = 1.602\times10^{-19}$ J	$k_B = 1.381\times10^{-23}$ J/K	Radiation: $t = \dfrac{2.42\text{ s}}{\sqrt{g_{\text{eff}}}}\left(\dfrac{\text{MeV}}{k_BT}\right)^2$	$m - M = 5\log(d) - 5$
$M_\odot = 1.989\times10^{30}$ kg	$\hbar = 6.582\times10^{-16}$ eV·s	**Temperature**	**Quark Charges**
$\text{y} = 3.156\times10^{7}$ s	$\hbar = 1.055\times10^{-34}$ J·s	$T_0 = 2.725$ K	Up: $\frac{2}{3}$, Down: $-\frac{1}{3}$
$G = 6.674\times10^{-11}$ m^3/kg/s^2	$\hbar c = 1.973\times10^{-7}$ eV·m		Strange: $-\frac{1}{3}$

Part III: Calculation [100/120 points] (20 each)
PHY 310: Answer **five** of the following six problems
PHY 610: Answer **all six** of the following problems

26. In homework, you showed that if matter/anti-matter segregation occurred, it could not have occurred at the time of quark confinement. If we came up with a mechanism, could it have occurred at the time of primordial nucleosynthesis, which occurred around k_BT = 80.0 keV?

$$\rho_{m0} = 2.674\text{ kg/Gm}^3,\quad \rho_{r0} = 7.80\times10^{-4}\text{ kg/Gm}^3,\quad \rho_{\Lambda0} = 5.947\text{ kg/Gm}^3.$$

(a) Estimate the age of the universe at this time. Use g_{eff} = 3.38.
(b) Find the temperature T in K and the red-shift factor 1+z at this time.
(c) The current density of ordinary matter is about $\rho_{b0} = 4.20\times10^{-28}\text{ kg/m}^3$. What would have been the density of matter at that time?
(d) Find the mass of a sphere of approximately the distance that objects could have moved at that time, if they were somehow swept together at nucleosynthesis, in units of $M_\odot$. Could it be that our universe is units of matter/anti-matter at this scale?

27. After hydrogen recombination, there were still lithium atoms that hadn't claimed their last electron. Lithium recombination would have occurred around when k_BT = 0.100 eV.
(a) Find the temperature T and red-shift z at this time.
(b) Was this during the matter- or radiation-dominated era? How old was the universe?
(c) At present there are about 0.250 baryons/m^3. What was the density of baryons then? Multiply by 5×10^{-9} to find the number density of lithium then.
(d) For each unrecombined lithium atom, there should be a corresponding free electron, which scatters with photons with the Thomson cross-section $\sigma_T = 6.65\times10^{-29}\text{ m}^2$. Would the density of free electrons before lithium recombination have made the universe opaque?

28. Three nuclei are produced in primordial nucleosynthesis that are unstable, with mean lifetimes listed at right. (d = 86,400 s, y = 3.156× 10^7 s)
(a) Are all of these after the era of nucleosynthesis? Are they all in the matter- or radiation-dominated eras?
(b) What is the temperature k_BT at each of these times? If you need it, use g_{eff} = 3.38.

Nucleus	Lifetime	k_BT (eV)
n^0	880 s	
^{3}H	17.77 y	
^{7}Be	76.78 d	

Units and Constants

$\text{pc} = 3.086\times10^{16}\ \text{m}$

$\text{eV} = 1.602\times10^{-19}\ \text{J}$

$M_\odot = 1.989\times10^{30}\ \text{kg}$

$\text{y} = 3.156\times10^{7}\ \text{s}$

$G = 6.674\times10^{-11}\ \text{m}^3/\text{kg/s}^2$

Physical Constants

$k_B = 8.617\times10^{-5}\ \text{eV/K}$

$k_B = 1.381\times10^{-23}\ \text{J/K}$

$\hbar = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$

$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s}$

$\hbar c = 1.973\times10^{-7}\ \text{eV}\cdot\text{m}$

Age of Universe

Matter

$$t = \frac{17.3\ \text{Gyr}}{(z+1)^{3/2}}$$

Radiation

$$t = \frac{2.42\ \text{s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_B T}\right)^2$$

Distance / Magnitudes

$d = 10^{1+\frac{m-M}{5}}\ \text{pc}$

$m - M = 5\log(d) - 5$

Quark Charges

Up: $\frac{2}{3}$, Down: $-\frac{1}{3}$

Strange: $-\frac{1}{3}$

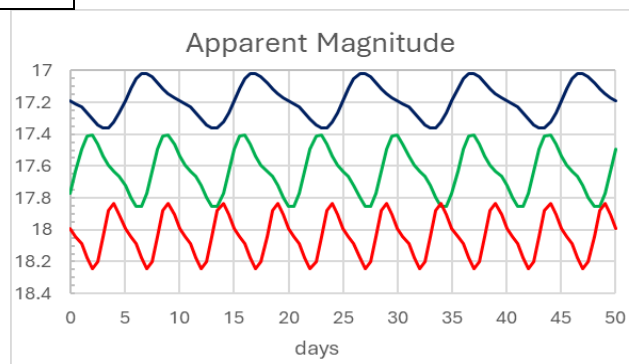
Cepheid Variables

$M = -2.81\cdot\log(P) - 1.43$

Temperature

$T_0 = 2.725\ \text{K}$

29. Three Cepheid Variable stars have their apparent magnitude measured as a function of time, as graphed at right.
    (a) For each of these stars, estimate the period and corresponding absolute magnitude
    (b) Estimate the apparent magnitude for each of the stars (defined as the average).
    (c) Find the approximate distance to all three stars.



30. The bottom quark has an estimated mass of about 4300 MeV/c^2.
    (a) At what approximate temperature $k_B T$ does the bottom quark disappear?
    (b) At right is a list of all the particles that are lighter or equal to the bottom quark, together with their number of spin states g and their spins. Find the value of g_{eff} at this time (include the bottom quark).
    (c) Estimate the age of the universe at this time. A bottom quark has a decay rate of about $\Gamma = 7.7\times10^{11}\ \text{s}^{-1}$; is this decay fast enough to keep the bottom in equilibrium?

Particles	g	spin
ν_1, ν_2, ν_3	3×2	½
e, μ, τ	3×4	½
u, d, s, c, b	5×12	½
photon	2	1
gluon	16	1

31. The Ξ is a particle with strangeness –2 that almost always decays as Ξ → $\Lambda^0\ \pi^-$ and *never* decays to Ξ → $\Lambda^0\ \pi^-\ \pi^0$. The spins and other properties of some of these particles are listed at right.

Part.	Type	spin	strange	mass
Λ^0	baryon	½	–1	1116 MeV/c^2
π^0	meson	0	0	135 MeV/c^2
π^-	meson	0	0	139 MeV/c^2

   (a) Is the Ξ a baryon, an anti-baryon, or a meson? Is it a fermion or a boson?
   (b) What is the charge of the Ξ?
   (c) Based on the decay that does occur and the one that doesn't, make reasonable guess on the minimum and maximum mass of the Ξ.
   (d) This boson lasts an average of 0.29 ns, which is a long time for such a particle. Why is it so long?
   (e) From the charge and strangeness, deduce the quark composition of Ξ.