Physics 310/610 – Cosmology

Homework Set J

1. In class, we demonstrated that for flat rotation curves, $V^2 = V_0^2$, the rotational and epicycle angular velocity are related by $\kappa = \Omega\sqrt{2}$. Consider instead the following two cases:
   (a) What is the formula for the rotation velocity around a point source of gravity of mass M? Find, in this case, a simple ratio between κ and Ω, no more complicated than the one we found for flat rotation curves.
   (b) What is the formula for the rotation velocity inside a sphere with uniform density ρ? Find, in this case, a simple ratio between κ and Ω, no more complicated than the one we found for flat rotation curves.

2. For the Milky Way galaxy at the radius of the Sun, find the three frequencies Ω, ν, and κ (in Myr^{-1}) and the corresponding periods (in Myr) in the neighborhood of the Sun. Assume we are 8.18 kpc from the center, that our galaxy has a flat rotation curve with $V_0 = 220$ km/s and the local mass density is $\rho = 0.07\ M_\odot \text{pc}^{-3}$. In one orbit of the galaxy, T_ϕ, how many cycles of up and down motion does the Sun undergo?

Note: Problem 3 requires that you do problem 2 first.

3. Assume the Sun is currently at $z = 0$ and $R = R_0$. The former implies that the Sun is right in the galactic plane (it's pretty close), the latter implies that the Sun is currently moving at exactly the right radial velocity for a circular orbit (it is not, but this simplifies things).
   (a) If the Sun is currently moving upwards at 8 km/s, determine the maximum distance z_0 (in pc) the Sun will reach above the plane. Also determine how long from now (in Myr) it will reach this position.
   (b) If the Sun is currently moving inwards at 11 km/s, determine the maximum amount ΔR (in pc) that it will drift in from its current distance of 8.18 kpc. Also determine how long from now (in Myr) it will reach this position.

Physics 610: Only do this problem if you are in the graduate version of this course

4. For vertical motion, we assumed the density was of the form $\rho = \rho_0$. A more realistic expression would be $\rho = \rho_0 e^{-|z|/h_z}$.
   (a) For this expression, find an expression for the scalar potential $\Phi(z)$ as a function of z. For simplicity, assume $\Phi(0) = 0$.
   (b) Suppose a star is bobbing up and down, reaching maximum height $z = \pm h$. Using conservation of energy, find the velocity v as a function of its height z and maximum height h.
   (c) Find an integral form for the time it takes to go through one-fourth of a cycle, from $z = 0$ to h, then quadruple it to get the total period T, as a function of h. The corresponding formula for uniform density would be $T = \dfrac{2\pi}{\sqrt{4\pi G\rho_0}}$.
   (d) Evaluate the integral in part (c) numerically for $h = \frac{1}{2}h_z$, $h = h_z$, and $h = 2h_z$. Compare to the result for uniform density.