Physics 310/610 – Cosmology

Homework Set Q

1. In this problem we are going to estimate the number of neutrinos in the current universe.
   (a) For each of the types of neutrino, find the current number density in m^{-3} using the neutrino temperature we found in class.
   (b) The neutrinos are known to have a small difference in mass, which we will treat as zero. One of them is known to be lighter than about 2 eV/c^2. Assuming this is the mass of all three neutrinos, find the total mass density (in kg/m^3) for all three neutrinos.
   (c) Find a maximum value for the neutrino contribution Ω_ν from neutrinos. Use the central value of the Hubble constant, $H_0 = 67.8$ km/s/Mpc.

2. Electrons and their anti-particles can be created from photon collisions, $\gamma\gamma \to e^+e^-$, with a cross section of approximately

$$\sigma \approx \left(\frac{\alpha\hbar c}{E}\right)^2$$

where $\alpha = \frac{1}{137}$ is the fine structure constant, and E is the typical energy of the two photons. However, this process only occurs if the energy is sufficient to make them, so that $E > mc^2$. The *real* formula is more complicated, but this is a good approximation.
   (a) Assume the photons have a typical energy $E = 3k_BT$. What is the minimum temperature required to start pair creating?
   (b) What is the age of the universe at this time, in s? Use $g_{\text{eff}} = 3.36$.
   (c) What is the approximate cross-section at this energy?
   (d) What is the number density of photons at this time? Use the formula

$$n = \frac{2\zeta(3)}{\pi^2}\left(\frac{k_BT}{\hbar c}\right)^3 = 0.24\left(\frac{k_BT}{\hbar c}\right)^3$$

   (e) What is the collision rate Γ for photon pairs? Assume the relative velocity is approximately $\Delta v = c$.
   (f) What is the number of collisions Γt? Will this process be in thermal equilibrium?

Graduate Problem: Do this problem only if you are in PHY 610.

3. In problem 1, you used the current temperature of the neutrinos to find the current number density. But if neutrinos are massive, they will not be in a thermal distribution.
   (a) Suppose there was a time when the neutrinos were effectively massless with a temperature T_ν. What is the typical energy for a single neutrino at such a temperature? What is the corresponding momentum p_ν?
   (b) Define "T_ν" at later times so it continues to drop $\propto a^{-1}$, even when the neutrino mass can no longer be neglected (so T_ν doesn't mean temperature any more). Argue that $p_\nu \propto T_\nu$ even after this (hint – what is the wavelength for a particle with momentum *p*?)
   (c) Estimate the momentum of a typical neutrino now.
   (d) The heaviest neutrino has a mass between 0.05 eV/c^2 and 2 ev/c^2. Based on these two limits, find a range of typical velocities for these neutrinos. Compare with escape velocity from our galaxy (about 600 km/s).