Physics 310/610 – Cosmology

Homework Set U

1. There is an additional problem with neutrinos as dark matter: It turns out the Pauli Exclusion Principle makes it hard to fit them in a galaxy. The computation is a bit complex, but we can approximate it pretty well as follows:
   (a) The local density of dark matter in the neighborhood of the Sun is around $0.4\ \mathrm{GeV}/c^2/\mathrm{cm}^3$. Assuming all three neutrinos weigh $3.7\ \mathrm{eV}/c^2$ each, what is the local number density n of neutrinos in m^{-3}? Then divide by three to get the number density of just one of the three types of neutrinos.
   (b) Suppose I placed each neutrino in a box of volume L^3. What would be the size of the box such that the density of neutrinos would match part (a)?
   (c) According to quantum mechanics, to fit a particle in a box of size L would require it to acquire a momentum of $p = \pi\hbar/L$. Work out the corresponding momentum. The most convenient units for this would be eV/c.
   (d) What is the corresponding velocity of the neutrinos? Compare to the approximate escape velocity of a galaxy like ours, probably around 600 km/s.

2. Suppose the universe is filled with matter in some regions, and anti-matter in others. Let's imagine the segregation occurred at the time of quark confinement, so somehow the quarks congregated together, and so did the anti-quarks. How large could these regions be?
   (a) Assuming the quarks and anti-quarks are gathered at the speed of light, what is the largest region *at the time* over which quarks could be gathered together?
   (b) Assuming the universe has kept aT constant since then (a fair approximation), how much has the universe grown since then? How large is this region now?
   (c) Find the baryonic mass of spheres of size given in part (b) using the density of baryons today $\rho_{b0} = 4.196\times10^{-28}\ \mathrm{kg/m}^3$.
   (d) As a basis of comparison, look up the Earth's mass. Is it plausible that this mechanism might produce pockets of matter as observed today?

Graduate Problems: There are no problems for PHY 610 on this homework