Physics 310/610 – Cosmology

Homework Set V

1. In class we found that the annihilation cross-section required to get the correct density of dark matter today is $\sigma \approx 4.5\times10^{-40}\ \text{m}^2$.
   (a) Assume the cross section takes a typical electromagnetic cross-section value, $\sigma = \alpha^2\hbar^2/\left(m^2c^2\right)$, where $\alpha = \frac{1}{137}$ is the fine structure constant. What would the relevant mass m in GeV/c^2 be?
   (b) Suppose the particle annihilates via weak interactions, with cross-sections of order $\sigma = G_F^2E_1E_2/\left(\hbar c\right)^4$, where $G_F/\left(\hbar c\right)^3 = 1.166\times10^{-5}\ \text{GeV}^{-2}$. What mass would you need? Keep in mind that the particles are non-relativistic when they collide.
   (c) There is an approximate maximum cross-section for annihilation $\sigma_{\max} = 4\pi\hbar^2/p^2$, where p is the momentum of the particles. Assume the kinetic energy of a typical dark-matter particle at freezeout is $\frac{3}{2}k_BT$, and that $k_BT = \frac{1}{30}mc^2$. What is the maximum mass that the dark matter particles could have and keep their cross section below $\sigma_{\max}$?

2. The universe contains a lot of energy, but it grew a lot during inflation. How much energy was there before?
   (a) The energy in radiation is given by

$$u = g_{eff}\frac{\pi^2\left(k_BT\right)^4}{30\left(\hbar c\right)^3}$$

   Calculate this energy density in J/m^3 both today ($g_{\text{eff}} = 3.36$) and at the end of inflation ($k_BT = 10^{16}$ GeV, and we'll guess $g_{eff} = 200$).
   (b) The current size of the visible universe is a sphere of radius about 13.5 Gpc. Convert this to meters. To a very good approximation, the scale factor is related to the size of the universe by the approximate relation $a \propto T^{-1}g_{eff}^{-1/3}$. What was the size of the region that become the *current* visible universe at the end of inflation?
   (c) Find the total radiation energy of the universe now and at the end of inflation (in J).
   (d) According to inflation, the universe grew in size by at least a factor of 10^{28}. Assuming the temperature at the start of inflation was the same as at the end, calculate the energy at the start of inflation. Then work out the equivalent mass, using $E = mc^2$. You should find that the mass required is remarkably small.

Graduate Problems: There are no problems for PHY 610 on this homework