December 10, 2025 Name ________________

Solutions to Final Exam PHY 310/610

This test consists of three parts. In parts II and III, PHY 310 students can skip one question of those offered, while PHY 610 students must answer all questions.

Part I: Multiple Choice (mixed new and review questions) [40 points] (2 points each)
PHY 310/610: For each question, choose the best answer

1. The first Friedmann equation says that $\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2}$. What is the quantity a?
   A) The distance to a particular galaxy, such as Andromeda
   B) The wavelength of a photon from the cosmic microwave background
   C) The distance you have to go to completely circle the universe
   D) The typical size of gaps between galaxies
   E) The "scale factor", to which all other distances are proportional

2. As the universe expands, radiation scales approximately as what power of a?
   A) a^{-4} B) a^{-3} C) a^{-2} D) $a^0 = 1$ E) a^4

3. The spectrum of the cosmic microwave background is what type of spectrum?
   A) Black body B) White noise C) Monochromatic D) Top Hat E) None of these

4. The isotope that is a "bottleneck" that prevents further steps of primordial nucleosynthesis until it happens is
   A) ^{3}He B) ^{4}He **C) Deuterium, ^{2}H** D) Tritium ^{3}H E) None of these

5. Which of the following do we <u>not</u> have a clear indication of why it is the way it is?
   A) Why the universe is almost homogenous
   B) Why the universe is almost isotropic
   C) Why the primordial abundance of Helium is about 25% by mass
   D) Why there is apparently more matter than anti-matter in the universe
   E) Why the cosmic microwave background has the spectrum it has

6. If you were to summarize general relativity in one to two sentences, what would it be?
   A) Matter causes other matter to accelerate, including relativistic effects
   B) Matter makes spacetime accelerate, and spacetime makes matter accelerate
   C) All curvature is illusion, which can be eliminated by working in accelerating coordinates
   D) Matter tells space how to curve, and space tells matter how to move
   E) Gravity causes objects to stop following geodesics, due to gravitational forces

7. When a galaxy collides with another galaxy, the most damage is done if the galaxy it collides with is ________ and the collision is ________ .
   A) Massive, fast **B) Massive, slow** C) Light, fast D) Light, slow E) I don't know

8. The reason there are giant elliptical galaxies often near the center of galaxy clusters is, we think, because
   A) The most massive galaxies attracted all the other galaxies in the cluster
   B) Gravitational lensing from the cluster makes it look bigger than it is
   C) These galaxies expelled their dark matter, but compensated with more ordinary matter
   D) Many galaxies merged to make these most massive galaxies
   E) These galaxies have expelled all the matter that would make it look dimmer

9. What event caused the photons to end up at a higher temperature than the neutrinos?
   A) Nucleosynthesis
   B) Proton/neutron freezeout
   C) Quark confinement
   D) Electroweak unification
   E) Electron-positron annihilation

10. The name of our galaxy is
    A) Milky Way B) Virgo C) Laniakea D) Coma E) Fornax

11. In Grand Unified theories (GUTs), the three forces of the standard model are unified into one at some very high energy. What is an additional likely prediction of GUTs?
    A) Neutrinos are massless
    B) Protons, even in hydrogen, are unstable
    C) Black holes should evaporate very slowly
    D) Dark matter is probably GUT scale particles
    E) None of these are common predictions of GUTs

12. Which of the following is <u>not</u> a name given to the potential shape of the universe due to curvature?
    A) Flat **B) Hyperbolic** C) Closed D) Open E) All of these are possibilities

13. Which of the following happened before proton/neutron freezeout?
    A) Recombination
    B) Matter/radiation equality
    C) Nucleosynthesis
    D) Quark confinement
    E) First structure

14. The age of the universe is approximately
    A) 13.8 Myr B) 138 Myr C) 1.38 Gyr D) 12.3 Gyr **E) 13.8 Gyr**

15. If we go back to very high red-shift, say $z = 10^4$, what do we think the value of Ω was?
    A) 0.2607 B) 0.9999 **C) 1.0000** D) 1.0001 E) Unknown

16. Strong interactions can violate conservation of
    A) Energy B) Baryon Number C) Electric Charge D) Strangeness **E) None of these**

17. Which two forces are already combined in standard model particle physics?
    A) Electromagnetism and gravity
    B) Electromagnetism and the weak force
    C) The strong force and the weak force
    D) The strong force and electromagnetism
    E) Gravity and the weak force

18. The primary evidence for the presence of large quantities of dark matter in spiral galaxies comes from
    A) Noticing how dark they are
    B) Obscuration of galaxies behind them
    C) Rotation curves that do not fall off with distance
    D) Photons coming from annihilation of dark matter
    E) Comparing the mas of individual stars to the amount of baryonic matter in them

19. Which force is *not* included in the standard model of particle physics
    A) Weak B) Strong **C) Gravity** D) Electromagnetism E) These are all included

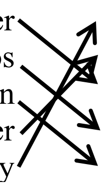
20. At modest red-shift, say $z = 4$, which component dominated the universe?
    A) Matter B) Radiation C) Neutrinos D) Dark energy E) Curvature

Part II Short Answer [40 points/50 points] (10 points each)
PHY 310: Choose **four** of the following five questions and give a short answer (1-3 sentences)
PHY 610: Answer **all five** questions

21. Hubble's law says that the velocity of a galaxy (as measured by its red-shift) is directly proportional to its distance. Give at least three reasons why this might be unreliable for very nearby galaxies of very distant galaxies.

For nearby galaxies the peculiar velocities of the galaxies can overwhelm the overall Hubble flow, so Hubble's law doesn't work. For very distant galaxies, we have a plethora of problems: distance is ill-defined (distance now? Distance then? Distance travelled by light?); also, the distance as measured can be influenced by the flashlight effect, where objects moving away from you tend to send most of their light that direction, and also the curvature of spacetime can mess up apparent distances. Finally, if you are looking very far away, you are looking so far into the past that in fact the expansion of the universe was likely a different rate than it is now.

22. At right are the five biggest components of the universe, and to its right are listed our best guess of Ω_i that each contributes. Match the items to their Ω_i values. You can just draw lines if you want.

Items		Ω_i
Ordinary matter		0.6889
Neutrinos		0.2607
Electromagnetic radiation		0.0490
Dark matter		0.0014
Dark energy		0.0001

23. Was the universe always transparent? If so, explain why, or if not, explain which step in the universe's evolution caused a transition (and was it transparent → opaque or vice versa). Don't just give the name of this step, describe what happened at that time.

The universe is mostly transparent now, just because most electrons are either low density or bound in atoms. But in the early universe, they were much higher density, and the temperature was so hot that they were not bound in atoms. Recombination is the process where free electrons pair up with protons to make hydrogen, which is much more transparent than electrons, so the transition opaque → transparent occurred at recombination

24. Inflation is a hypothesized stage in the universe when the universe grew at an exponential rate in a very short time. Describe which three things inflation is supposed to explain.

The universe appears to be homogenous and isotropic; this can be explained if very distant points started in causal contact but then were pushed to very large separations. The universe appears to be almost perfectly flat; this can be explained if the universe expanded so much the curvature became negligible. Finally, inflation takes tiny regions where quantum fluctuations occur, and expands them to cosmic sizes, which can account for the fluctuations in the microwave background (and ultimately the formation of galaxies).

25. List at least four future events that we discussed might occur of a catastrophic nature (*i.e.*, it could wipe out humanity)

We will be (1) hit by a giant asteroid some time in the next 100 million years or so. The Sun will grow (2) brighter enough to cause catastrophic global warming in about 1 billion years, then it will become a (3) red giant, probably destroying all life on Earth. Even if we travel to another star, eventually there will be no more stars being born (4), and eventually even the oldest stars will die (5). It is likely that the proton decays, in which case all matter disappears eventually (6). The only thing that remains will be black holes, but even if we can tap their energy, eventually they will all die. (7)

Part III: Calculation [100/120 points] (20 each)
PHY 310: Answer **five** of the following six problems
PHY 610: Answer **all six** of the following problems

26. In homework, you showed that if matter/anti-matter segregation occurred, it could not have occurred at the time of quark confinement. If we came up with a mechanism, could it have occurred at the time of primordial nucleosynthesis, which occurred around k_BT = 80.0 keV?

$$\rho_{m0} = 2.674 \text{ kg/Gm}^3, \quad \rho_{r0} = 7.80\times10^{-4} \text{ kg/Gm}^3, \quad \rho_{\Lambda 0} = 5.947 \text{ kg/Gm}^3.$$

(a) Estimate the age of the universe at this time. Use g_{eff} = 3.38.

This temperature is well above the matter/radiation equality, so we are forced to use the radiation formula. We know that g_{eff} = 3.38 is approximately right for anything after electron positron annihilation (which occurs around k_BT = 170 keV), so we have

$$t = \frac{2.42 \text{ s}}{\sqrt{3.38}}\left(\frac{\text{MeV}}{0.080\,\text{MeV}}\right)^2 = 206 \text{ s}.$$

The number given in class was 200 s, so this is pretty much the same.

(b) Find the temperature T in K and the red-shift factor 1+z at this time.

We use the formulas

$$T = \frac{k_BT}{k_B} = \frac{8.00\times10^4 \text{ eV}}{8.617\times10^{-5} \text{ eV/K}} = 9.28\times10^8\text{K},$$
$$1+z = \frac{T}{T_0} = \frac{9.28\times10^8 \text{ K}}{2.725 \text{ K}} = 3.407\times10^8.$$

There is very little point in adjusting this to find z instead.

(c) The current density of ordinary matter is about $\rho_{b0} = 4.20\times10^{-28} \text{ kg/m}^3$. What would have been the density of matter at that time?

Normal matter scales proportional to $(1+z)^3$, so the density at the time would have been

$$\rho = \rho_{b0}(1+z)^3 = \left(4.20\times10^{-28} \text{ kg/m}^3\right)\left(3.407\times10^8\right)^3 = 0.0166 \text{ kg/m}^3.$$

(d) Find the mass of a sphere of approximately the distance that objects could have moved at that time, if they were somehow swept together at nucleosynthesis, in units of $M_\odot$. Could it be that our universe is units of matter/anti-matter at this scale?

At most, matter could have gathered from a distance of approximately *ct*, where *c* is the speed of light and *t* is the age. This distance is

$$d = ct = (206\text{ s})(2.998\times10^{8}\text{ m/s}) = 6.176\times10^{10}\text{ m}.$$

To find the mass gathered, we simply multiply the density times the volume, which yields

$$M = V\rho = \frac{4\pi}{3}d^{3}\rho = \frac{4\pi}{3}(6.176\times10^{10}\text{ m})^{3}(0.0166\text{ kg/m}^{3}) = 1.639\times10^{31}\text{ kg}$$
$$= \frac{1.639\times10^{31}\text{ kg}}{1.989\times10^{30}\text{ kg}} = 8.24\,M_{\odot}$$

There are stars more massive than this, and they certainly aren't a mix of matter and anti-matter. We also see colliding clouds of gas MUCH larger than this, and we see cosmic rays coming from throughout our galaxy that seem to be primarily matter, so it's not plausible that the universe has matter/anti-matter divisions on this scale, even if we could think of a mechanism to cause this.

27. After hydrogen recombination, there were still lithium atoms that hadn't claimed their last electron. Lithium recombination would have occurred around when k_BT = 0.100 eV.

(a) Find the temperature T and red-shift z at this time.

This is straightforward. We have

$$T = \frac{k_BT}{k_B} = \frac{0.100\text{ eV}}{8.617\times10^{-5}\text{ eV/K}} = 1160\,\text{K},$$
$$1+z = \frac{T}{T_0} = \frac{1160}{2.725\text{ K}} = 426.$$

(b) Was this during the matter- or radiation-dominated era? How old was the universe?

The rule of thumb is that it is matter dominated when $z < 3000$, so this works well. We use the formula

$$t = \frac{17.3\text{ Gyr}}{(z+1)^{3/2}} = \frac{17.3\times10^{9}\text{ yr}}{426^{1.5}} = 1.968\times10^{6}\text{ yr} = 1.968\text{ Myr}.$$

(c) At present there are about 0.250 baryons/m^3. What was the density of baryons then? Multiply by 5×10^{-9} to find the number density of lithium then.

The number density, like the mass density, scales as $(1+z)^3$, so the number density of baryons is about

$$n = n_0(1+z)^{3} = (0.25\text{ m}^{-3})(426)^{3} = 1.933\times10^{7}\text{ m}^{-3}.$$

However, the density of lithium was much lower (I kind of faked the number) so we have

$$n_{Li} = \left(5\times10^{-9}\right)n = \left(5\times10^{-9}\right)\left(1.933\times10^{7}\ \text{m}^{-3}\right) = 0.0966\ \text{m}^{-3}.$$

(d) For each unrecombined lithium atom, there should be a corresponding free electron, which scatters with photons with the Thomson cross-section $\sigma_T = 6.65\times10^{-29}\ \text{m}^2$. Would the density of free electrons before lithium recombination have made the universe opaque?

The reaction rate is $\Gamma = n_{Li}\sigma\left(\Delta v\right)$, where Δv is the relative velocity. Since the photon moves at c and the atom is almost at rest, we have

$$\Gamma = n_{Li}\sigma\left(\Delta v\right) = \left(0.0966\ \text{m}^{-3}\right)\left(6.65\times10^{-29}\ \text{m}^2\right)\left(2.998\times10^{8}\ \text{m/s}\right) = 1.93\times10^{-21}\ \text{s}^{-1}.$$

To see if an average photon scatters, we then multiply this by the age of the universe, to get

$$\Gamma t = \left(1.93\times10^{-21}\ \text{s}^{-1}\right)\left(1.968\times10^{6}\ \text{yr}\right)\left(3.156\times10^{7}\ \text{s/yr}\right) = 1.20\times10^{-7}.$$

Since most photons will have no interactions, the universe is effectively transparent, and we were right to ignore Lithium

28. Three nuclei are produced in primordial nucleosynthesis that are unstable, with mean lifetimes listed at right. (d = 86,400 s, y = 3.156× 10⁷ s)

Nucleus	Lifetime	k_BT (eV)
n^0	880 s	34,900
^{3}H	17.77 y	48.5
^{7}Be	76.78 d	445

(a) Are all of these after the era of nucleosynthesis? Are they all in the matter- or radiation-dominated eras?

Nucleosynthesis takes place about 200 s after the big bang, so they are all after the era of nucleosynthesis. Matter-radiation equality occurred many thousands or years later, so they are all in the radiation era.

(b) What is the temperature k_BT at each of these times? If you need it, use g_{eff} = 3.38.

In each case, we solve for the temperature using the radiation dominated formula, which we rearrange to give

$$\left(\frac{k_BT}{\text{MeV}}\right)^2 = \frac{2.42\ \text{s}}{t\sqrt{g_{\text{eff}}}}.$$

We now just apply it in each case, converting all times to seconds. We should probably add the time when nucleosynthesis ends, around 200 s, to each of these times, which only makes a difference for the neutron.

$$\left(\frac{k_B T_n}{\text{MeV}}\right)^2 = \frac{2.42\ \text{s}}{(880\ \text{s}+200\ \text{s})\sqrt{3.38}} = 0.00121,$$

$$k_B T_n = \sqrt{0.00121}\ \text{MeV} = 0.0349\ \text{MeV} = 34{,}900\ \text{eV},$$

$$\left(\frac{k_B T_{3H}}{\text{MeV}}\right)^2 = \frac{2.42\ \text{s}}{(17.7\ \text{y})(3.156\times10^7\ \text{s/y})\sqrt{3.38}} = 2.356\times10^{-9},$$

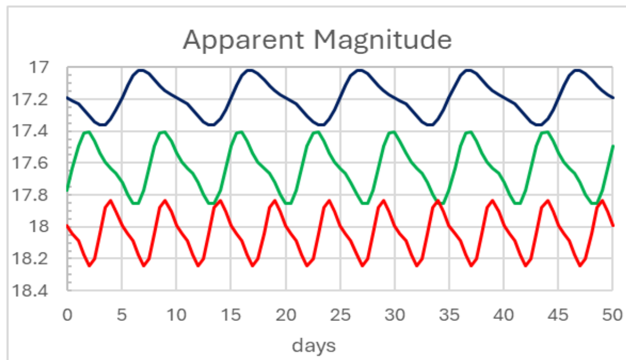
$$k_B T_{3H} = \sqrt{2.356\times10^{-9}} = 4.85\times10^{-5}\ \text{MeV} = 48.5\ \text{eV},$$

$$\left(\frac{k_B T_{7Be}}{\text{MeV}}\right)^2 = \frac{2.42\ \text{s}}{(76.78\ \text{d})(8.64\times10^4\ \text{s/d})\sqrt{3.38}} = 1.98\times10^{-7},$$

$$k_B T_{3H} = \sqrt{1.98\times10^{-7}} = 4.45\times10^{-4}\ \text{MeV} = 445\ \text{eV},$$

29. Three Cepheid Variable stars have their apparent magnitude measured as a function of time, as graphed at right.

(a) For each of these stars, estimate the period and corresponding absolute magnitude



Dr. Carlson was kind enough to provide the formula relating period to absolute magnitude. You have to measure the period in days. Looking at it, we see that the red star seems to repeat every five days, the green one has six peaks in 42 days, and the bule one seems to repeat every ten days. So we have

$$P_r = 5.0\ \text{d},\quad M_r = -2.81\cdot\log(P) - 1.43 = -2.81\cdot\log(5.0) - 1.43 = -3.39,$$

$$P_g = 7.0\ \text{d},\quad M_g = -2.81\cdot\log(P) - 1.43 = -2.81\cdot\log(7.0) - 1.43 = -3.80,$$

$$P_b = 10.0\ \text{d},\quad M_r = -2.81\cdot\log(P) - 1.43 = -2.81\cdot\log(10.0) - 1.43 = -4.24.$$

(b) Estimate the apparent magnitude for each of the stars (defined as the average).

We are again fortunate the Dr. arlson reminded us to use the average. Keeping in mind that the numbers increase downwards, the red one seem to be averaging around $m_r = 18.03$,the green one around $m_g = 17.62$, and the blue one around $m_b = 17.20$.

(d) Find the approximate distance to all three stars.

We simply use the same formula in all three cases:

$$d_r = 10^{1+(m_r - M_r)/5} \text{ pc} = 10^{1+(18.03+3.39)/5} \text{ pc} = 192 \text{ kpc},$$
$$d_g = 10^{1+(m_g - M_g)/5} \text{ pc} = 10^{1+(17.62+3.80)/5} \text{ pc} = 192 \text{ kpc},$$
$$d_b = 10^{1+(m_b - M_b)/5} \text{ pc} = 10^{1+(17.20+4.24)/5} \text{ pc} = 194 \text{ kpc}.$$

I was trying to get all three of them to be at the same distance, and did pretty well.

30. The bottom quark has an estimated mass of about 4300 MeV/c^2.

(a) At what approximate temperature $k_B T$ does the bottom quark disappear?

Particles	g	spin
ν_1, ν_2, ν_3	3×2	½
e, μ, τ	3×4	½
u, d, s, c, b	5×12	½
photon	2	1
gluon	16	1

It happens around

$$k_B T = \tfrac{1}{3} mc^2 = \tfrac{1}{3}(4300 \text{ MeV}) = 1430 \text{ MeV}.$$

(b) At right is a list of all the particles that are lighter or equal to the bottom quark, together with their number of spin states *g* and their spins. Find the value of g_{eff} at this time (include the bottom quark).

We simply add up all the spin states *g*, but we multiply the fermions (the neutrinos, charged leptons, and quarks) by an extra factor of $\frac{7}{8}$. This gives us

$$g_{eff} = g_B + \tfrac{7}{8} g_F = 2 + 16 + \tfrac{7}{8}(3\times 2 + 3\times 4 + 5\times 12) = 86.25.$$

(c) Estimate the age of the universe at this time. A bottom quark has a decay rate of about $\Gamma = 7.7\times 10^{11}\ \text{s}^{-1}$; is this decay fast enough to keep the bottom in equilibrium?

The age of the universe at this temperature is

$$t = \frac{2.42 \text{ s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_B T}\right)^2 = \frac{2.42 \text{ s}}{\sqrt{86.25}}\left(\frac{1}{1430}\right)^2 = 1.27\times 10^{-7} \text{ s}.$$

To figure out if decay (and reverse decay, or fusion) can keep it in equilibrium, we multiply the decay rate by the age of the universe, to yield

$$\Gamma t = \left(7.7\times 10^{11} \text{ s}^{-1}\right)\left(1.27\times 10^{-7} \text{ s}\right) = 9.77\times 10^{4}.$$

Since it has many chances to undergo such a decay, this keeps it in equilibrium, though in fact there are other processes that are even faster.

31. The Ξ is a particle with strangeness –2 that almost always decays as Ξ → Λ^0 π^- and *never* decays to Ξ → Λ^0 π^- π^0. The spins and other properties of some of these particles are listed at right.

Part.	Type	spin	strange	mass
Λ^0	baryon	½	–1	1116 MeV/c^2
π^0	meson	0	0	135 MeV/c^2
π^-	meson	0	0	139 MeV/c^2

(a) Is the Ξ a baryon, an anti-baryon, or a meson? Is it a fermion or a boson?

There is one fermion on the right (the Λ^0) and again one baryon on the right (the Λ^0), so the Ξ must be a baryon and a fermion.

(b) What is the charge of the Ξ?

The charge on the right is -1, so it must be charge -1. The actual name of this particle is the Ξ^- (there is another particle called the Ξ^0).

(c) Based on the decay that does occur and the one that doesn't, make reasonable guess on the minimum and maximum mass of the Ξ.

Both of the decays violate strangeness, which is a sign they are both weak decays, but they both have the same charge, etc., on the right, so the main thing that distinguishes them is that the masses on the right are different. The two-particle decay has a total mass

$$m_{\Lambda 0} + m_{\pi-} = \left(1116 \text{ MeV}/c^2\right) + \left(139 \text{ MeV}/c^2\right) = 1255 \text{ MeV}/c^2 .$$

We know therefore that the Ξ^- is heavier than this. For the other decay, the mass would be

$$m_{\Lambda 0} + m_{\pi-} + m_{\pi 0} = \left(1116 \text{ MeV}/c^2\right) + \left(139 \text{ MeV}/c^2\right) + \left(135 \text{ MeV}/c^2\right) = 1390 \text{ MeV}/c^2 .$$

This suggests (but does no prove) that it might be lighter than this, which would make this decay impossible. We therefore have

$$1255 \text{ MeV}/c^2 < m_{\Xi-} < 1390 \text{ MeV}/c^2$$

The mass is actually 1322 MeV/c^2, but you have no way of knowing that.

(d) This boson lasts an average of 0.29 ns, which is a long time for such a particle. Why is it so long?

The decay is slow because strangeness is violated, which means it is a weak interaction.

(e) From the charge and strangeness, deduce the quark composition of Ξ.

It's a baryon, so it has three quarks. Each strange quark has strangeness –1, so we must have two strange quarks. To get the charge to come out right, the other quark must also have charge -1/3, so the quark content is [ssd].

Units and Constants

$\text{pc} = 3.086\times10^{16}\ \text{m}$

$\text{eV} = 1.602\times10^{-19}\ \text{J}$

$M_\odot = 1.989\times10^{30}\ \text{kg}$

$\text{y} = 3.156\times10^{7}\ \text{s}$

$G = 6.674\times10^{-11}\ \text{m}^3/\text{kg/s}^2$

Physical Constants

$k_B = 8.617\times10^{-5}\ \text{eV/K}$

$k_B = 1.381\times10^{-23}\ \text{J/K}$

$\hbar = 6.582\times10^{-16}\ \text{eV}\cdot\text{s}$

$\hbar = 1.055\times10^{-34}\ \text{J}\cdot\text{s}$

$\hbar c = 1.973\times10^{-7}\ \text{eV}\cdot\text{m}$

Age of Universe

Matter

$$t = \frac{17.3\ \text{Gyr}}{(z+1)^{3/2}}$$

Radiation

$$t = \frac{2.42\ \text{s}}{\sqrt{g_{\text{eff}}}}\left(\frac{\text{MeV}}{k_B T}\right)^2$$

Distance / Magnitudes

$$d = 10^{1+\frac{m-M}{5}}\ \text{pc}$$

$$m - M = 5\log(d) - 5$$

Quark Charges

Up: $\frac{2}{3}$, Down: $-\frac{1}{3}$

Strange: $-\frac{1}{3}$

Cepheid Variables

$$M = -2.81\cdot\log(P) - 1.43$$

Temperature

$T_0 = 2.725\ \text{K}$