PHY 310/610

Solutions to Midterm Exam
October 8, 2025

This test consists of three parts. For the first and second parts, you may write your answers directly on the exam, if you wish. For the other parts, use separate sheets of paper.

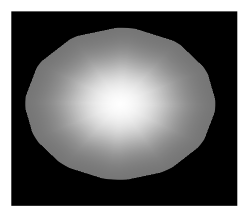
Part I: Multiple Choice [2 points each] Everyone: Answer all questions.
For each question, choose the best answer

1. Which spectral class of stars would be the hottest?
   A) F2 B) F7 C) K4 D) K6 E) M5

2. Which of the following elements counts towards the metallicity of a star, according to astronomers?
   A) Iron (only)
   B) Iron and calcium, but not silicon
   C) Iron, calcium, and silicon, but not oxygen
   D) Iron, calcium, silicon, and oxygen, but not helium
   E) Iron, calcium, silicon, oxygen, and helium

3. Sometimes behind a galaxy cluster we will see multiple galaxies with remarkably similar spectra. Why does this happen?
   A) It shows that there are very distant galaxies that form in clusters of identical galaxies
   B) There is one giant galaxy behind them, but dust blocks all but a few holes where the spectrum comes through
   C) One very bright galaxy is reflecting off of clouds that are orbiting it
   D) The galaxy is moving so fast it appears to be at several locations at once
   E) It is just a single galaxy with multiple images caused by gravitational lensing

4. The galaxy classification of the cartoon galaxy at right is approximately
   A) SAb B) SBc **C) E2** D) E7 E) Im



5. Black holes are not supposed to allow any light to escape, and yet we see X-rays coming from many AGN's. How is this possible?
   A) X-rays are actually coming from the accretion disk near the black hole
   B) X-rays are produced by Hawking radiation from black holes
   C) The jets from the center smash into the intergalactic medium, producing X-rays
   D) The black holes have attracted numerous O and B stars, which produce X-rays
   E) X-rays are not the same thing as light; they CAN escape from black holes

6. Which of the following would by itself give you an estimate of the temperature of a star?
   A) Luminosity B) Brightness C) Mass **D) Color** E) Age

7. We are approximately what distance from the center of our galaxy?
   A) 8.3 pc B) 83 pc **C) 8.3 kpc** D) 8.3 Mpc E) 83 Mpc

8. MACHOS (massive compact halo objects) sometimes pass between us and a distant star, causing the star behind it to
   A) Be dimmed or blocked by the obscuring intervening object
   B) Be made temporarily brighter, due to gravitational lensing
   C) Be gravitationally **red**-shifted due to the MACHO
   D) Be gravitationally **blue**-shifted due to the MACHO
   E) Gain extra absorption lines as the light passes through the foreground object

9. The way you can tell how old a cluster of stars is is by
   A) Measuring the metallicity of the cluster
   B) Counting the number of living vs. dead stars
   C) Measuring the amount of dust that dying stars have ejected
   D) Studying the concentration of helium vs. hydrogen in a typical star
   E) Looking at the Hertzsprung-Russell diagram and seeing where the turn off point is

10. Spectroscopic parallax is somewhat unreliable, even when applied to main sequence stars, because
   A) It is often difficult to determine the spectral class of a star
   B) The luminosity of main sequence stars varies somewhat based on age
   C) Stars that are red- or blue-shifted will have their spectral class apparently shifted
   D) Many stars are surrounded by clouds of gas that hide their true luminosity
   E) Most stars are binaries or multiples, and it is impossible to disentangle their spectra

11. The name of the nearest large galaxy (not ours) to ours is
   A) Andromeda B) Virgo C) Milky Way D) Laniakea E) Coma

12. Central dominant galaxies (cD galaxies) are giant elliptical galaxies normally found where?
   A) The center of small galaxy groups
   B) The edge of small galaxy groups
   C) The center of large galaxy clusters
   D) The edge of large galaxy clusters
   E) Isolated, with no other galaxies nearby

13. Which of the following objects would you likely use radar distancing to find the distance to?
   A) Andromeda galaxy, M31 (only)
   B) Venus (only)
   C) The Coma cluster (only)
   D) All of the above
   E) None of the above

14. Approximately what fraction of a typical galaxy cluster's mass is in the form of dark matter?
   A) 2% B) 15% C) 50% **D) 85%** E) 98%

15. The amount of light coming from a black body at temperature T is proportional to
   A) $T^{1/4}$ B) $T^{1/2}$ C) T D) T^2 **E)** $\boldsymbol{T^4}$

Part II: Short Answer PHY 310: Choose three of the four questions PHY 610: Answer all four questions. Write 2-4 sentences about each of the following [10 points each]

PHY 310: You may skip one question

16. Measuring the kinetic energy of a typical star in a galaxy is easy. Measuring the potential energy is hard. Explain how the virial theorem allows us to estimate the potential energy/gravitational potential, using at least one equation. When will the virial theorem tend to apply?

When the virial theorem is satisfied, the relationship $2E_K + E_P = 0$ is satisfied, and hence if you measure the kinetic energy you can also determine the potential energy. However, this equation is only true if the system has “virialized”; that is, gone through enough interactions for the energy to transform from one to the other. This typically occurs after order one orbit.

17. We would like to accurately know the distance to the Andromeda Galaxy (M31). Explain why using Hubble’s Law or Type Ia supernovae are probably not the methods we use.

Hubble’s Law describes the overall expansion of the universe, but for nearby objects, the general trend can be overwhelmed by peculiar velocities, which describe motions due to nearby gravitational interactions. M31 in particular is actually moving towards us. Type Ia supernovae are extremely rare, and in any given galaxy (M31), there probably hasn’t been one recently.

18. What is the main reason that we believe typical spiral and barred spiral galaxies have dark matter in them? Explain qualitatively how this is measured and how it shows dark matter.

You can study the shift of spectral lines at various distances from the center of galaxies, which allow you to deduce the orbital velocities. These velocities should fall off at large distances if dark matter is absent; the fact that the rotation curves are flat indicates the presence of dark matter.

19. In automobile crashes, it is worse if you collide with a more massive car and if you collide at high velocity. Of course, if you miss, even by a tiny amount, it has no consequences. Explain how galaxy collisions are like or unlike car crashes.

In galaxy collisions, colliding with a large galaxy is more destructive than colliding with a small galaxy, but because gravitational influences last longer the slower you collide, low speed collisions are more destructive than high speed collisions. Also, a near miss can still be disruptive, since gravitational interactions can still occur.

Physical Constants	Units	Brightness/Magnitude	Galactic Orbits
$k_B = 1.381\times10^{-23}$ J/K	$\text{pc} = 3.086\times10^{16}$ m	$F = 2.518\times10^{-8}\,\text{W/m}^2\left(10^{-\frac{2}{5}m}\right)$	$\Omega = \dfrac{V_0}{R_0}$
$\hbar = 1.055\times10^{-34}$ J·s	$\text{M}_\odot = 1.988\times10^{30}$ kg	**Cepheid Period/Luminosity**	$v = \sqrt{4\pi G\rho_0}$
$h = 6.626\times10^{-34}$ J·s	$\text{yr} = 3.156\times10^{7}$ s	$M = -2.43\log(P) - 1.62$	**Distance/Magnitude**
$G = 6.674\times10^{-11}\,\text{m}^3/\text{kg/s}^2$	$1'' = 4.848\times10^{-6}$ rad		$d = 10^{1+\frac{m-M}{5}}$ pc
			$m - M = 5\log(d) - 5$

Part III: Calculation: [20 points each]
PHY 310: Choose four of the five problems; PHY 610: Do all five problems.
For each of the following problems, find the answer, explaining your work.

20. Star *X* has apparent magnitude *m* = 8.40. Suddenly it brightens and becomes magnitude *m* = 6.89

(a) By approximately what factor has its flux increased?

Looking at the equations, we see that $F = k\cdot10^{-\frac{2}{5}m}$. It follows that

$$\frac{F_{\text{after}}}{F_{\text{before}}} = \frac{k\cdot10^{-\frac{2}{5}m_a}}{k\cdot10^{-\frac{2}{5}m_b}} = 10^{\frac{2}{5}(m_b-m_a)} = 10^{0.400(8.40-6.89)} = 10^{0.564} = 4.02.$$

(b) Maybe the star got brighter by somehow increasing its radius, while leaving its temperature the same. By what factor must its radius have increased?

The total flux is proportional to the luminosity, which is proportional to the area of the star's surface and temperature to the fourth power, $L = 4\pi\sigma R^2T^4$. Since the star is at the same distance, its luminosity presumably increased by the same amount. It follows that we must have

$$4.02 = \frac{L_{\text{after}}}{L_{\text{before}}} = \frac{4\pi\sigma R_a^2T_a^4}{4\pi\alpha R_b^2T_b^4} = \left(\frac{R_a}{R_b}\right)^2,$$

$$\frac{R_a}{R_b} = \sqrt{4.02} = 2.004.$$

It could have increased its radius by about a factor of about 2.

(c) Maybe the star got brighter by somehow increasing its temperature, while leaving its radius the same. By what factor must its temperature have increased?

The computation is the same, but this time the radius cancels out, so

$$4.02 = \frac{L_{\text{after}}}{L_{\text{before}}} = \frac{4\pi\sigma R_a^2T_a^4}{4\pi\alpha R_b^2T_b^4} = \left(\frac{T_a}{T_b}\right)^4,$$

$$\frac{T_a}{T_b} = 4.02^{1/4} = 1.416.$$

The temperature could have increased by a factor of about 1.4.

(d) Assuming one of the two assumptions (a) or (b) is right, what observations could tell you which is correct?

The peak of the spectrum, or even easier, the color of the star should shift towards hotter colors (blue) if the temperature changes, but not if just the radius changed.

21. The star Vega has a parallax of 0.1302″, a proper motion of 0.326″/yr, and the K-line spectral line normally at 393.366 nm occurs instead at 393.348 nm.

(a) Whas it the distance to Vega in pc?

We simply use the formula

$$d = \frac{1}{p} = 7.68 \text{ pc}.$$

(b) What is the radial velocity of Vega, in km/s? It it moving towards us or away from us?

The spectral shift is small, so we can use the formula $z = v_r/c$. We therefore have

$$v_r = cz = c\left(\frac{\lambda_0}{\lambda} - 1\right) = c\left(\frac{393.348}{393.366} - 1\right) = \left(-4.58\times10^{-5}\right)\left(299{,}800 \text{ km/s}\right) = -13.7 \text{ km/s}.$$

Since the result is negative, Vega is moving towards us.

(c) What is the transverse velocity of Vega, in km/s?

The hardest part of this is unit conversion. We can use the formula $v_t = \mu d$, but we have to put the angular speed in radians. We also have to convert years to seconds, and parsecs to m. We have

$$\begin{aligned} v_t = \mu d &= \left(0.326''/\text{yr}\right)\left(7.68 \text{ pc}\right)\frac{\left(4.848\times10^{-6} \text{ rad}/1''\right)\left(3.086\times10^{16} \text{ m/pc}\right)}{3.156\times10^{7} \text{ s/yr}} \\ &= 11{,}900 \text{ m/s} = 11.9 \text{ km/s}. \end{aligned}$$

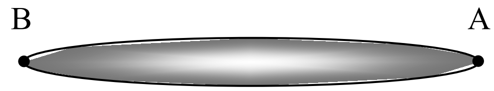
(d) What is the speed of Vega relative to the Sun, in km/s?

We simply add the transverse and radial velocities in quadrature; that is

$$v^2 = v_r^2 + v_t^2 = \left(-13.7 \text{ km/s}\right)^2 + \left(11.9 \text{ km/s}\right)^2 = 329 \text{ km}^2/\text{s}^2,$$

$$v = \sqrt{329 \text{ km}^2/\text{s}^2} = 18.1 \text{ km/s}.$$

22. At right is an image of a spiral galaxy, exactly edge on to us. The atomic spectral feature normally at a wavelength of 21.206 cm on Earth has a wavelength of λ_A = 21.283 cm on the right side and λ_B = 21.247 cm on the left.



(a) Find the radial velocity relative to us at points A and B

The spectral shifts at both sides of the galaxy are relatively small, so again we can use the approximation $z = v_r/c$. We therefore have

$$v_A = cz_A = c\left(\frac{\lambda_{0A}}{\lambda} - 1\right) = c\left(\frac{21.283}{21.206} - 1\right) = (0.00363)(299{,}800 \text{ km/s}) = 1089 \text{ km/s},$$

$$v_B = cz_B = c\left(\frac{\lambda_{0B}}{\lambda} - 1\right) = c\left(\frac{21.247}{21.206} - 1\right) = (0.00193)(299{,}800 \text{ km/s}) = 580 \text{ km/s}.$$

Since both results came out positive, both sides are moving away from us.

(b) Find the average velocity of the galaxy and the rotational velocity around its center. Which side is rotating towards us?

Averaging the two velocities tells us the velocity of the galaxy compared to us, which is about

$$v_{avg} = \frac{v_A + v_B}{2} = \frac{(1089 \text{ km/s}) + (580 \text{ km/s})}{2} = 834 \text{ km/s},$$

which of course is away from us. The rotational velocity is the difference between this and either of the others.

$$v_{rot} = \left|v_A - v_{avg}\right| = \left|v_B - v_{avg}\right| = 254 \text{ km/s}.$$

Since the left side is moving slower away from us, it must be rotating towards us.

(c) The radius of this galaxy 12.0 kpc. Find the mass of the galaxy within the region *AB* using this information.

The centripetal acceleration v_{rot}^2/R must match the gravitational acceleration, GM/R^2. Solving for M, we have

$$\frac{GM}{R^2} = \frac{v_{rot}^2}{R},$$

$$M = \frac{Rv_{rot}^2}{G} = \frac{(1.2\times10^4 \text{ pc})(2.54\times10^5 \text{ m/s})^2}{6.674\times10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}} \cdot \frac{3.086\times10^{16} \text{ m}}{\text{pc}}$$

$$= \frac{3.58\times10^{41} \text{ kg}}{1.988\times10^{30} \text{ kg/M}_\odot} = 1.80\times10^{11} \text{ M}_\odot$$

This is not terribly different from the mass of our galaxy.

23. An alien race in a distant galaxy has determined that they are currently at a distance R = 9.70 kpc from the center of their galaxy, and z_0 = 85.0 pc above the plane of their galaxy. They are orbiting the center at V_0 = 247 km/s. The local density of their plane is $\rho = 0.081\, M_{\odot}\text{pc}^{-3}$.

(a) Find the frequency Ω for their motion around the center and their orbital period in Myr.

This is mostly a matter of unit conversion. We use the formula

$$\Omega = \frac{V_0}{R_0} = \frac{2.47\times10^5 \text{ m/s}}{9700 \text{ pc}} \cdot \frac{1}{3.086\times10^{16} \text{ m/pc}} = \left(8.25\times10^{-16} \text{ s}^{-1}\right)\left(3.156\times10^7 \text{ s/yr}\right)$$
$$= 2.60\times10^{-8} \text{ yr}^{-1},$$
$$T_\phi = \frac{2\pi}{\Omega} = \frac{2\pi}{2.60\times10^{-8} \text{ yr}^{-1}} = 2.41\times10^8 \text{ yr} = 241 \text{ Myr}.$$

This is a little slower than we go around our galaxy, because it is circling in a larger radius.

(b) Find their frequency for vertical oscillations ν and the period for this motion in Myr.

We first convert the density into SI units:

$$\rho_0 = \frac{\left(0.081 M_{\odot}\text{pc}^{-3}\right)\left(1.988\times10^{30}\,\text{kg}/M_{\odot}\right)}{\left(3.086\times10^{16} \text{ m/pc}\right)^3} = 5.48\times10^{-21} \text{ kg/m}^3 .$$

We then substitute this into the equation

$$\nu = \sqrt{4\pi G\rho_0} = \sqrt{4\pi\left(6.674\times10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}\right)\left(5.48\times10^{-21} \text{ kg/m}^3\right)}$$
$$= \left(2.144\times10^{-15} \text{ s}^{-1}\right)\left(3.156\times10^7 \text{ s/yr}\right) = 6.77\times10^{-8} \text{ yr}^{-1},$$
$$T_z = \frac{2\pi}{\nu} = \frac{2\pi}{6.77\times10^{-8} \text{ yr}^{-1}} = 9.29\times10^7 \text{ yr} = 92.9 \text{ Myr}.$$

(c) The aliens find that their star currently has no vertical motion, v_{z0} = 0. We said in class that the formula for their vertical motion would be $z = z_0 \sin\left(\nu t\right)$. Explain how this isn't exactly right and how to fix it.

As discussed in class, the motion can be a sine or cosine, or even a combination of these. If it is a sine, its initial position would be $z = 0$ and its velocity would be non-zero, but for a cosine, the initial position would be at a maximum and its initial vertical velocity would be zero. So the correct formula is $z = z_0 \cos\left(\nu t\right)$.

24. Three galaxies each are found to have a Cepheid variable start with a period of P = 67 days. The apparent magnitudes of each of these is measured, denoted m (CV).

Gal.	m (CV)	m (RG)	d (kpc)	M (RG)
A	16.97	18.93	403	–4.10
B	18.51	20.47	820	–4.10
C	12.68	24.63	5600	–4.11

(a) What is the absolute magnitude of these Cepheid Variable stars?

We use the standard formula for Cepheid Variable stars, namely

$$M = -2.43\log(P) - 1.62 = -2.43\log(67) - 1.62 = -6.06.$$

(b) What is the distance in kpc of each of these galaxies?

We use the standard distance/magnitude relationship

$$d_A = 10^{1+\frac{1}{5}(m_A - M)}\text{ pc} = 10^{1+0.2(16.97+6.06)}\text{pc} = 4.03\times10^5\text{ pc} = 403\text{ kpc},$$

$$d_B = 10^{1+\frac{1}{5}(m_B - M)}\text{ pc} = 10^{1+0.2(18.51+6.06)}\text{pc} = 8.20\times10^5\text{ pc} = 820\text{ kpc},$$

$$d_A = 10^{1+\frac{1}{5}(m_C - M)}\text{ pc} = 10^{1+0.2(22.68+6.06)}\text{pc} = 5.60\times10^6\text{ pc} = 5600\text{ kpc}.$$

(c) For the same three galaxies, the brightest red giant's magnitude is also measured. In each case, estimate the absolute magnitude of the brightest red giant.

We simply use the same formulas in reverse, namely $m - M = 5\log(d) - 5$, which we solve for M:

$$M_A = m_A - 5\log(d_A) + 5 = 18.93 - 5\log(4.03\times10^5) + 5 = -4.10,$$

$$M_B = m_B - 5\log(d_B) + 5 = 20.47 - 5\log(8.20\times10^5) + 5 = -4.10,$$

$$M_C = m_C - 5\log(d_C) + 5 = 21.63 - 5\log(1.40\times10^6) + 5 = -4.11.$$

(d) Argue based on your previous computations that the brightest red giant is or is not a good standard candle.

The fact that it keeps coming out with the same number, namely, M = –4.10, indicates that this is a good standard candle. In fact, this is absurdly accurate, but that's because your professor fudged the numbers.