Physics 712
Chapter X Problems

1. A 6 volt battery is connected between two parallel metal plates of area 100 cm^2 and separation 1 mm, initially both neutral.
   (a) How much charge flows when the battery is connected?
   (b) With the battery still connected, the plates are moved to 0.5 mm separation. What is the change in electrostatic energy stored in the field?
   (c) How much energy was supplied by the battery as the plates were moved?
   (d) Starting over with the 1 mm gap and the plates at 6 volt potential difference, disconnect the battery. Find the work required by an external force (e.g. you) to insert a slab of dielectric slab of permittivity $\varepsilon = 2\varepsilon_0$ between the plates. Is the slab pulled in by the capacitor, or must it be pushed in?

2. Consider a model in which all electrons are bound approximately in a damped harmonic oscillator, such that their displacement **x** in the presence of an electric field will be governed by $m\ddot{\mathbf{x}} = -e\mathbf{E} - m\gamma\dot{\mathbf{x}} - m\omega_0^2\mathbf{x}$. Assuming the positions and electric field both proportional to $e^{-i\omega t}$, find the relationship between **E** and **x**. Then find the polarization $\mathbf{P} = -ne\mathbf{x}$, and the complex permittivity ε. As a check, make sure you get the same answer as we did for a collisionless plasma ($\gamma = \omega_0 = 0$).

3. What is the real part of the permittivity of a material if

$$\mathrm{Im}\left[\varepsilon(\omega)\right] = \begin{cases} a\left(\omega_0^2\omega - \omega^3\right) & \text{for } \omega < \omega_0 , \\ 0 & \text{for } \omega > \omega_0 ? \end{cases}$$

   It should be noted that you will not have to use the principal part when attempting to find $\mathrm{Re}\left[\varepsilon(\omega)\right]$ if $\omega > \omega_0$.

4. The conductivity of copper is $5.96\times10^7\ \Omega^{-1}\mathrm{m}^{-1}$. Suppose power is being transmitted at 60 Hz along a high-voltage wire. What is the skin depth δ in copper at this frequency?

5. Suppose you are at sufficiently high frequency that you can treat the free electrons in a conductor as a plasma.
   (a) Find a formula for the index of refraction if $\omega < \omega_p$. Show that, at all angles and for both polarizations, the intensity of the reflected light matches that of the incident light.
   (b) For normal incidence and $\omega < \omega_p$, find a formula for how the intensity drops off in the conductor as a function of distance within the conductor.
   (b) Assume that in aluminum, there is one conduction electron per atom. Find the plasma frequency for aluminum. For visible light with vacuum wavelength of 500 nm, how far into an aluminum mirror must you go before the power drops by a factor of 10^5?