Physics 780 – General Relativity

Homework C

7. Each of the following formulas is true for an appropriate value of *k* in flat 4D-spacetime. In each case, find *k*
   (a) $\eta_{\mu\nu}\eta^{\mu\nu} = k$
   (b) $\eta_{\mu\nu}\eta_{\alpha\beta}\eta^{\mu\gamma}\eta^{\beta\alpha}\delta^{\nu}_{\gamma} = k$
   (c) $\tilde{\varepsilon}_{\mu\nu\alpha\beta} = k\tilde{\varepsilon}^{\mu\nu\alpha\beta}$
   (d) $\tilde{\varepsilon}_{\mu\nu\alpha\beta}\tilde{\varepsilon}^{\mu\nu\alpha\beta} = k$
   (e) $\tilde{\varepsilon}_{\mu\nu\alpha\beta}\eta^{\mu\nu} = k\eta_{\alpha\beta}$

8. This problem has to do with Maxwell's equations
   (a) Show that Maxwell's first equation, $\partial_{\nu}F^{\mu\nu} = J^{\mu}/\varepsilon_0$, automatically assures that current is conserved, $\partial_{\mu}J^{\mu} = 0$.
   (b) It is common to write the electromagnetic field tensor in the form $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, where A_{μ} is the four-vector potential. Show that if you do this then the second Maxwell equation is automatically satisfied.

9. A particle of charge *q* and mass *m* is initially moving with velocity $\mathbf{v} = (v_1, 0, v_2)$. It is placed in a region with a uniform magnetic field in the *z*-direction $B_3 = B$.
   (a) What is the initial four-velocity $U^{\mu}(\tau = 0)$?
   (b) Write down differential equations for all four components of the four velocity $dU^{\mu}/d\tau$. Solve these equations, subject to the initial conditions, for U^0 and U^3.
   (c) Find a second order differential equation for U^2 of the form $\frac{d^2}{d\tau^2}U^2 = -\omega^2 U^2$. What is ω?
   (d) Solve the equation for part (c), subject to the initial conditions. There should be one unknown parameter describing U^2 at this point.
   (e) Using the formula for $dU^2/d\tau$, find a formula for U^1. By matching the initial conditions, you should now have all components of U^{μ} as a function of τ.