Physics 780 – General Relativity

Homework D

10. The stress-energy tensor for a perfect fluid is given by $T^{\mu\nu} = (\rho + p)U^\mu U^\nu + p\eta^{\mu\nu}$. All substances we know of have positive energy density, that is, $T^{00} > 0$.
    (a) If perfect fluid is going at almost the speed of light, what condition on ρ and/or p will assure that $T^{00} > 0$. This is called the *null energy condition*.
    (b) What condition on ρ and/or p will assure that $T^{00} > 0$ if the fluid is at rest (this is trivial)? Argue that if both the condition from (a) and (b) are true, then $T^{00} > 0$ at *all* speeds. This is called the *weak energy condition*.
    (c) There is a special relationship between ρ and p that makes the stress-energy tensor independent of the fluid's "speed". What is that relationship (it's easier than it sounds). Vacuum energy density acts this way.
    (d) Radiation has the property that the trace of the stress energy tensor $T = \eta_{\mu\nu}T^{\mu\nu} = 0$. What is the relationship between ρ and/or p in this case?

11. The electromagnetic field produces a stress-energy tensor $T^{\mu\nu} = \varepsilon_0 \left(F^{\mu\lambda} F^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu} F^{\lambda\sigma} F_{\lambda\sigma} \right)$.
    (a) Show that for this stress-energy tensor, the trace is always $T = \eta_{\mu\nu}T^{\mu\nu} = 0$.
    (b) Suppose you have a uniform electric field in the *x*-direction, $F^{01} = -F^{10} = E$. Find all non-vanishing components of the stress-energy tensor in this case.