Physics 780 – General Relativity

Homework Set R

43. This problem has a lot to do with units. The goal is to keep careful track of them.
   (a) Working in SI units, if you have a charge q, what is the electric field at a distance r? Compare with our formula for the electric field for the Reissner-Nordström. At least at large r, they should be the same. Based on this, find a formula relating Q to q.
   (b) You're not done with units! Because we are working in general relativity, there can easily be some factors of c, the speed of light hidden in your formula for part (a). Given that GQ^2 has units of m^2, revise your formula from part (a) by adding an appropriate power of c to the relationship you found there.
   (c) At large distances, we can use classical formulas to calculate forces. Supposed a black hole of mass M and charge q is so charged up that a proton with mass m and charge e far from the black hole feels exactly balancing forces from gravity and electromagnetism. What is the ratio q/M for this black hole in C/kg? You can use classical formulas, since we are far from the black hole.
   (d) For the black hole in part (c), find the value of $Q/\left(M\sqrt{G}\right)$. You may have to include factors of c to make this expression dimensionless.

44. Although the Kerr metric is not diagonal, it *is* diagonal on the z-axis, $\theta = 0$. By symmetry, any object moving along the z-axis will continue moving along the z-axis.
   (a) Find the metric and the inverse metric on the z-axis as a function of r. Ignore the $g_{\phi\phi}$ part of the metric. As a check, you should find that $g^{tt} = -g_{rr}$.
   (b) As usual, since we have a time translation symmetry, ∂_t, there will be a conserved component of the four-velocity, whose value we will call $-E$. If an object starts at rest from infinity, what is E?
   (c) By demanding that $U^\mu U_\mu = -1$, find a formula for U^r as a function of r for an object that starts at rest at infinity. Note that we are assuming $U^\phi = U^\theta = 0$.
   (d) Find a formula for the time it takes to fall from a distance r to $r = 0$. It will be an integral that you probably can't do.
   (e) Even if you can't do the integral, convince yourself that it is finite. I did it by setting $r = ax^2$, and then convincing myself that the resulting integral was finite for any finite r.