Physics 780 – General Relativity

Homework Set S

45. Imagine we have an empty universe, so $\rho = 0$.
    (a) Using the first Friedmann equation, what must be the value of k? Solve for $a(t)$ as a function of time, choosing the constant of integration so that $a(0) = 0$.
    (b) Write the full metric. We have just discovered a new metric, different from flat space, with nothing in it! Or have we? Big hint: look at problem set F, problem 16.

46. Suppose the universe is flat ($k = 0$) and is filled with a fluid of just one type with $\rho \propto a^{-n}$, with $n > 0$. I recommend writing $\frac{8}{3}\pi G\rho = Ca^{-n}$, where C is constant.
    (a) Using the first Friedmann equation, write a formula of the form $dt = f(a)\,da$, where $f(a)$ is a simple formula. Integrate it to get a formula for the age of the universe t in terms of a, defining $t = 0$ as the time when $a = 0$.
    (b) Using the fact that H_0 is the current value of $\dot{a}/a$, find a formula for the current age of the universe *just* in terms of H_0 and n.
    (c) The current value of the age of Hubble's constant is $H_0 = 67.7$ km/s. Find the value of H_0^{-1}, called the *Hubble time*, in Gyr.
    (d) Assuming we have matter ($n = 3$) or radiation ($n = 4$), based on parts (b) and (c) how old is the universe in each case? Compare to the age of the oldest stars, somewhere around 13 Gyr.